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A Comparative Study of Three Diagnostic Arithmetic Tests

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A COMPARATIVE STUDY
of Three
DIAGNOSTIC ARITHMETIC TESTS

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FOREWORD

This study was undertaken with a view to making a comparison of the results obtained by the use of different arithmetic tests. Those chosen for comparison were the Cleveland Survey Tests, the Woody Scale, and the Monroe Diagnostic Tests. All three of these purport to be diagnostic in their nature, and if this be true they should lead to approximately the same conclusions concerning the arithmetical abilities of the children tested. It was with a desire to determine whether they do this or not that this study was made.

The study is divided into two parts. Part I gives a discussion of the value of arithmetic tests in general and a description of the tests used. Part II gives the results obtained by giving the three different tests to a group of children and the conclusion reached from these results.

A Comparative Study of Three Diagnostic Arithmetic Tests

PART I

In recent years there has been a most remarkable development of all kinds of educational tests and measurements. Of course it has always been necessary for teachers to measure their pupils' attainments in some fashion or other. Some children were promoted at the end of the year while others were retained in the same grade. This was done because the teacher judged that in the one case sufficient progress had been made to enable the children to do the work of the next grade, while in the other such progress had not been made. In order to arrive at these conclusions the teacher had to measure the achievements of the various children in the grade. Again at the end of each month teachers were called upon to "grade" the pupils in the various subjects that they happened to be studying. This again called for the measuring process. But the sort of measuring done was of a very indefinite kind. It was made up very largely of the teacher's estimates of the child, and into it entered a great many things besides the ability to do certain specific things. Then, too, the teacher's knowledge of the specific abilities of the children was exceedingly limited. It is true so-called tests and examinations were given but they were of such a nature as to test the abilities of the children only in a very general way. In fact they were often said to test the children's general ability in this, that, or the other subject, whereas, as we now know, there is no such thing as general ability in a subject. There are, in fact, as many separate abilities in even a single subject as there are different types of mental activities involved.

Another difficulty with these tests was that they lacked uniformity. If a child did not do as well in a test in arithmetic this week as he did last week it was taken to mean that he was losing ground. This might not be at all true. The tests were different and therefore there was really no basis for comparison. Again, if a child in the sixth grade got a grade of 90% in arithmetic while one in the eighth grade got a grade of 70% this fact did not give any basis for comparing the abilities of these two children. Their grades were obtained upon entirely different tests.

This then was the state of things up to within the last twenty years.

At the present time, however, quite a different state of affairs obtains. Tests and scales have been developed and standardized so that a teacher need no longer be in doubt about how her pupils compare with other pupils in the same grade, with pupils in other grades of the same school, with pupils in other school systems, or, best of all, with their own previous records in any specific ability.

The Courtis Standard Research tests were not given in this experiment, but as all of the scales have been built, to a greater or less extent, upon them they will be discussed here.

Inspired by the work of Rice and Stone, the pioneers in the field of tests and measurements in arithmetic, Mr. C. A. Courtis took up the task of developing a set of standard tests. He worked out a set, now known as series A, which he gave to thousands of children in different parts of the country. Five thousand children were tested in Detroit; 33,000 in New York; 20,000 in Boston, and many others in smaller systems. In scoring these papers perhaps the most remarkable fact brought out was the wide range of variability shown by the children in any given grade. Some children in the sixth grade,

for instance, made scores lower than the average of the third grade while others exceeded the average of the eighth grade. In spite of this fact, however, Mr. Courtis found that the scores for the children of the sixth grade tended to be grouped about a certain standard of excellence which was a little lower than that about which the scores of the seventh grade children tended to be grouped and higher than that of the fifth grade. This lead to the establishment of certain standards of excellence for the different grades in the particular abilities tested by these examples.

Series A of the Courtis tests includes eight separate tests, each one containing more examples than the swiftest child could complete in the time allotted. The tests are thus a measure of speed as well as of accuracy. These eight tests take up the combinations in addition, subtraction, multiplication and division, speed copying of figures, one-step reasoning problems, abstract examples in the four fundamentals and two-step reasoning problems.

After using this series for several years Mr. Courtis, and others as well, found that it was not satisfactory in several respects. In the first place it was too expensive in both time and money. Then again it did not given an adequate test of the abilities most needed by the pupil. It tested the pupil's knowledge of the addition combinations but did not give much information concerning his ability to apply this knowledge to the addition of columns of numbers. The same is true of the other operations. He found also that there was practically no relation between a child's ability to give the addition combinations and his ability to add a long column of figures. He therefore devised a second group of tests known as series B. This group consists of four tests, one for each of the fundamental operations.

Test 1 involves the addition of columns of 9 three-place numbers; Test 2 the subtraction of eight-place from eight- and nine-place numbers; Test 3 the multiplication of four- by two-place numbers, and Test 4 the division of four- and five-place numbers by two-place numbers. These tests have also been thoroughly standardized.

These Courtis tests are of great value to the teacher or supervisor of arithmetic. They furnish an instrument by means of which he may determine the degree of excellence reached by a grade or an individual in any one of the four fundamental operations. But they are not primarily diagnostic in their nature. Whatever diagnosis is made by their use is general and not specific in its nature. They do show, for instance, that a certain grade is low in addition, but they give no suggestion as to just which one of the several abilities required in addition is at fault. Then, too, they are limited to the field of the four fundamental operations with integers.

Realizing these facts a number of investigators have been at work devising tests that would be primarily diagnostic in their aim. Three such tests or scales have been devised and used to a considerable extent, viz., the Cleveland Survey tests, the Woody scale, and the Monroe tests. We shall consider them in the order given.

THE CLEVELAND SURVEY TESTS

When Dr. Judd and his co-laborers started the Cleveland Survey they looked over the field of existing tests and scales in arithmetic and decided that none of those that had been developed up to that time would meet the needs of the situation. The Courtis tests seemed to be the most promising but they were open to serious objections.

Series A they felt to be unsatisfactory for the same reasons as those already given in this discussion. Series B used as a supplement to series A would constitute a decided improvement. But even this combination did not go far enough to suit them. By using the combination they saw that they could measure general attainment in each of the four fundamental operations but nothing more. In other words the test would not be diagnostic. For

instance, a pupil might show by his work on Test 1, Series A, that he knew his addition tables perfectly, and yet he might fail utterly on Test 1 of Series B. These facts, they argued, would be worth knowing, but they would be of comparatively little value unless supplemented by other facts. The question of why he failed on the second test would remain unanswered. It might be because he failed "to bridge the attention spans," or because of his inability to "carry," but the tests would give no indication as to which it was. In order to throw light upon this question it was necessary to introduce between the simple types of the first series and the more complex types of the second some intermediate forms.

These investigators accordingly secured the co-operation of Mr. Courtis and worked out what are known as the Cleveland Survey Tests in Arithmetic. These tests are here reproduced in full. They consist of 15 sets, designated A, B,——O. There are four sets in addition (A, E, J, M), two in subtraction (B, F), three in multiplication (C, G, L), four in division (D, I, K, N), and two in fractions (H, O). This gives a spiral arrangement, as the pupil begins with Set A and takes each set in its proper order.

In the sets involving addition, Set A, which is simply Test 1 of Series A in the Courtis Standard tests, requires simply a knowledge of the combinations. Set E requires the addition of columns of five one-place numbers. This, then, is a new type. The pupil must combine the first two numbers and must then hold this sum in mind while he combines it in turn with the next number. Set J requires the addition of 13 one-place numbers. This again introduces a new element, "bridging the attention span." It is a well known fact that the addition of a long column of numbers is not one continuous process. The individual rather adds up several numbers, pauses for a moment while the attention wavers, then continues the addition. The fourth set, M, requires the addition of columns of five four-place numbers. This brings in another mental process, that of "carrying." The four sets then indicate ability or lack of ability (1) in addition combinations, (2) in adding several numbers in a column, (3) in "bridging the span of attention," and (4) in "carrying."

The tests contain but two sets in subtraction. Set B tests the knowledge of the subtraction combinations, while set F, the subtraction of three- from three- and four-place numbers, tests a knowledge of borrowing. This covers the field of subtraction.

In multiplication there are three sets. Set C gives the simple combinations, Set G, the multiplication of four-place by one-place numbers, tests a knowledge of "carrying," while set L, the multiplication of four- by two-place numbers, requires a knowledge of the mechanics of handling the multiplication by a second number in the multiplier and of the addition of the partial products.

In division there are again four tests. Set D tests a knowledge of the simple combinations. Set I, the division of five- by one-place numbers, introduces "carrying." Set K, the division of three- and four- by two-place numbers, brings in the simplest type of long division, involving no carrying in the multiplication, and no borrowing in the subtraction. Set N is the more complex type of division requiring both carrying and borrowing.

These tests attempt also to diagnose the pupil's ability in fractions in addition to his ability in the fundamentals with integers. For this purpose Sets H and O were introduced. Set H requires addition and subtraction of fractions having a common denominator, while in Set O fractions of unlike denominators are added, subtracted, multiplied and divided.

The Cleveland Survey tests carry out the plan of the Courtis Standard tests as to time allowance. The time limit ranges from 30 seconds to 3 minutes. The plan was to give sufficient time for even the slowest pupil to work out at least one example but not enough to allow the swiftest to finish them all.

Arithmetic Exercises

Cleveland Survey Tests

Name ..... Age today .....
Years Months

Grade ..... School ..... Room .....

Teacher ..... Date today .....

Have you ever repeated the arithmetic of a grade because of non-promotion or transfer from other school? If so, name grade .....
Explain cause .....

.....

.....

Inside this folder are examples which you are to work out when the teacher tells you to begin. Work rapidly and accurately. There are more problems in each set than you can work out in the time that will be allowed. Answers do not count if they are wrong.

Begin and stop promptly at signals from the teacher.

	A	B	C	D	E	F	G	H
A								
R								
Rank								

	I	J	K	L	M	N	O	
A								
R								
Rank								

SET A—Addition

													Ats.	Rts.
1	6	9	0	4	1	7	9	3	2	1	3	6		
2	6	5	1	2	3	7	6	0	4	5	8	9		
0	3	8	9	7	8	2	1	4	8	0	2	3		
7	2	1	9	6	0	5	6	7	9	5	7	1		
4	7	0	3	1	2	5	6	7	5	8	6	9		
6	9	8	5	4	9	8	0	2	1	3	5	0		
4	2	9	7	4	5	7	4	8	0	3	9	2		
3	2	3	8	0	2	1	9	6	0	4	1	8		
5	0	6	2	4	5	1	6	3	7	9	0	4		
7	4	3	1	8	9	0	2	3	4	8	6	5		

SET B—Subtraction—

9	7	11	8	12	1	9	13	4	12		
9	3	6	1	3	0	7	8	3	6		
8	11	12	5	10	6	11	15	10	12		
0	9	7	1	2	0	7	8	9	4		
2	7	13	3	10	1	6	15	4	8		
1	5	7	2	5	1	3	9	2	3		
4	10	13	10	9	5	8	17	6	11		
4	7	5	1	4	5	6	9	4	8		
5	12	15	5	16	7	8	16	9	11		
0	9	6	3	8	0	5	7	1	4		

SET C—Multiplication—

3	4	9	0	5	4	2	7	4	9		
2	7	8	2	6	1	9	6	0	5		
9	5	4	7	6	2	3	9	0	7		
1	2	8	0	5	1	3	6	5	4		
1	2	7	0	8	7	3	9	2	4		
6	8	7	6	3	1	8	9	0	3		
1	4	8	0	4	1	6	8	0	9		
5	4	9	3	5	4	2	8	7	3		
1	3	6	0	3	2	6	7	5	4		
7	4	8	0	9	2	3	9	5	6		

SET D—Division—							Ats.	Rts.
3)9	4)32	6)36	2)0	7)28	9)9	3)21		
6)48	1)1	5)10	2)6	4)24	7)63	6)0		
8)32	1)8	5)30	8)72	1)0	9)36	1)7		
2)10	7)42	1)1	6)18	3)6	4)20	7)49		
1)3	2)8	6)6	3)27	8)64	1)2	4)16		
5)0	3)24	9)63	2)4	8)24	7)7	2)18		
6)42	3)0	7)21	4)4	3)15	9)81	7)0		

SET E—Addition—

5	2	9	2	6	1	4	9
2	8	8	8	3	4	6	7
2	8	0	5	4	2	5	1
0	5	7	0	8	5	3	5
4	1	6	6	8	4	4	3
6	2	6	8	5	4	1	3
7	7	2	5	9	0	4	7
8	3	3	1	6	8	1	2
5	4	9	3	3	5	8	9
5	1	3	8	8	5	4	6

SET F—Subtraction—

616	1248	1365	1092	716
456	709	618	472	344
1267	1335	707	816	1157
509	419	277	335	908
1355	908	519	1236	1344
616	258	324	908	818
1009	768	1269	615	854
269	295	772	527	286

SET G—Multiplication—

2345	9735	8642	6789	2345
2	5	9	2	6
9735	2468	6789	3579	2468
9	3	6	3	7
5432	9876	8642	3579	9876
4	8	5	7	4
5432	3689	2457	9863	7542
8	5	6	4	7

SET H—Fractions—

Ats.	Rts.

$\frac{3}{5}+\frac{1}{5}=$	$\frac{6}{9}-\frac{4}{9}=$	$\frac{4}{9}+\frac{1}{9}=$	$\frac{8}{9}-\frac{7}{9}=$
$\frac{1}{9}+\frac{5}{9}=$	$\frac{3}{7}-\frac{1}{7}=$	$\frac{1}{7}+\frac{4}{7}=$	$\frac{6}{7}-\frac{2}{7}=$
$\frac{2}{9}+\frac{4}{9}=$	$\frac{4}{5}-\frac{1}{5}=$	$\frac{5}{8}+\frac{1}{8}=$	$\frac{6}{9}-\frac{5}{9}=$
$\frac{7}{9}+\frac{1}{9}=$	$\frac{5}{7}-\frac{2}{7}=$	$\frac{5}{9}+\frac{2}{9}=$	$\frac{8}{9}-\frac{1}{9}=$
$\frac{1}{8}+\frac{3}{8}=$	$\frac{6}{8}-\frac{1}{8}=$	$\frac{2}{7}+\frac{1}{7}=$	$\frac{5}{9}-\frac{4}{9}=$
$\frac{2}{9}+\frac{6}{9}=$	$\frac{4}{8}-\frac{3}{8}=$	$\frac{4}{7}+\frac{2}{7}=$	$\frac{7}{9}-\frac{5}{9}=$

SET I—Division—

$4\overline{)55424}$	$7\overline{)65982}$	$2\overline{)58748}$	$5\overline{)41780}$
$9\overline{)98604}$	$6\overline{)57432}$	$3\overline{)82689}$	$6\overline{)83194}$
$8\overline{)51496}$	$9\overline{)75933}$	$8\overline{)87856}$	$4\overline{)38968}$

SET J—Addition—

7	9	4	7	2	9	6	7	7	8	9	4	3	2
5	2	5	1	9	6	9	1	8	0	5	3	1	1
4	4	8	9	4	2	6	5	5	7	3	7	7	6
2	8	1	4	8	4	7	1	4	1	4	7	6	6
0	7	8	2	1	1	4	6	8	5	2	2	6	8
6	2	4	3	5	7	0	4	1	8	6	0	9	1
5	5	5	8	5	3	3	5	2	1	3	9	3	6
1	3	1	5	2	9	7	3	1	3	9	5	4	9
8	6	3	2	4	2	1	3	3	7	2	6	5	7
3	1	9	7	3	3	6	7	9	4	2	3	4	5
2	4	6	7	6	8	0	6	8	9	8	4	2	2
9	8	3	1	7	5	6	1	4	4	5	8	9	2
9	8	5	9	6	5	6	7	5	4	6	8	9	4

SET K—Division—

$21\overline{)273}$	$52\overline{)1768}$	$41\overline{)779}$	$22\overline{)462}$	$31\overline{)837}$
$42\overline{)966}$	$23\overline{)483}$	$72\overline{)1656}$	$81\overline{)972}$	$73\overline{)1679}$
$21\overline{)294}$	$62\overline{)1984}$	$31\overline{)527}$	$52\overline{)2184}$	$41\overline{)984}$
$32\overline{)384}$	$51\overline{)2397}$	$82\overline{)1968}$	$71\overline{)3692}$	$22\overline{)484}$
$41\overline{)1681}$	$33\overline{)693}$	$61\overline{)1586}$	$53\overline{)1166}$	$31\overline{)496}$

SET L—Multiplication—

8246	3597	5739	2648
29	73	85	46
4268	7593	6428	8563
37	64	58	207

SET M—Addition—

Ats.	Rts.

7493	8937	8625	2123	5142	3691
9016	6345	4091	1679	0376	4526
6487	2783	3844	5555	4955	7479
7591	4883	8697	6331	9314	2087
6166	1341	7314	6808	5507	8165
5226	9149	6268	9397	7337	8243
2883	8467	7725	6158	2674	6429
2584	0251	8331	3732	9669	9298
0058	7535	5493	4641	5114	7404
2398	5223	3918	7919	8154	2575

SET N—Division—

$67\overline{)32763}$	$48\overline{)28464}$	$97\overline{)36084}$	$59\overline{)29382}$
$78\overline{)69888}$	$88\overline{)34496}$	$69\overline{)40296}$	$38\overline{)26562}$

SET O—Fractions—

$\frac{11}{15}+\frac{1}{6}=$	$\frac{9}{14}-\frac{1}{4}=$	$\frac{3}{5}\times\frac{5}{6}=$
$\frac{5}{6}-\frac{2}{21}=$	$\frac{5}{6}\times\frac{19}{20}=$	$\frac{11}{12}\div\frac{5}{8}=$
$\frac{1}{6}\times\frac{3}{10}=$	$\frac{5}{6}\div\frac{11}{15}=$	$\frac{5}{12}+\frac{2}{8}=$
$\frac{20}{21}\div\frac{1}{6}=$	$\frac{3}{4}+\frac{3}{18}=$	$\frac{3}{8}-\frac{3}{10}=$

Instructions for Examiners

Have the children fill out the blanks at the top of the first page. Have them start and stop work together. Let there be an interval of half a minute between the sets of examples. Take two days for the test; give down through I the first day, and complete the test on the next day. The time allowances below must be followed exactly.

Set A.....30 seconds	Set F..... 1 minute	Set K.....2 minutes
Set B.....30 seconds	Set G..... 1 minute	Set L.....3 minutes
Set C.....30 seconds	Set H.....30 seconds	Set M.....3 minutes
Set D.....30 seconds	Set I..... 1 minute	Set N.....3 minutes
Set E.....30 seconds	Set J..... 2 minutes	Set O.....3 minutes

Have the children exchange papers. Read the answers aloud and let the children mark each example that is correct, "C." For each set let them count the number of problems attempted and the number of "C's" and write the numbers in the appropriate columns at the right of the page.

The records should then be transcribed to the first page. Please verify the results set down by the pupils.

THE WOODY SCALES

The Woody scales are the results of another attempt to devise a series of tests for measuring achievements in the four fundamental operations of arithmetic. The author of the scales makes the statement that the fundamental aim was to devise a series which would indicate the type of problems and the difficulty of the problems that a class could solve correctly. Each test is, therefore, composed of as great a variety of problems as possible. They are arranged in the order of increasing difficulty, beginning with the easiest that can be found and gradually increasing in difficulty until the last can be solved by only a small per cent of the pupils in the eighth grade. The degree of difficulty of each problem was determined, not by analysis, but by submitting the tests to a large number of children and computing the difficulty of each problem from the number of children that were able to solve it.

In building the scales under the above outlined plan the author made up tests containing as great a variety of problems as possible and submitted them to a large number of children. The results of these tests showed that the preliminary tests did not conform to the plan adopted. They did not show an arrangement of problems such that they were solved by a gradually increasing per cent of the pupils from one grade to the next higher. There were large gaps between certain problems. These defects were remedied by introducing extra problems to fill up these gaps and by dropping out such problems as were solved by a higher percentage of pupils in the lower grades than in the higher grades.

This method of construction has been severely criticised. It is maintained that if we are to measure arithmetical abilities with any degree of certainty we must include in our tests problems that exercise all the important types of arithmetical abilities, whether or not this gives us a list of problems gradually increasing in difficulty. This criticism is undoubtedly just to a certain extent. At least it is safe to say that if we are to use the Woody scales intelligently we must know their limitations.

These scales are published in two series, A and B. Series A is the more complete, while series B is made from series A by leaving out part of the problems, and is intended to be used by those who can devote but a limited time to giving the tests. Series A was used in this study and is given here in full.

Series A

Addition Scale

By Clifford Woody

City..... County..... School..... Date.....
Name.....When is your next birthday?.....
How old will you be?.....Are you a boy or a girl?.....
In what grade are you?.....Teacher's name.....

(1)
2
3
—

(2)
2
4
3
—

(3)
17
2
—

(4)
53
45
—

(5)
72
26
—

(6)
60
37
—

(7) $3 + 1 =$

(8) $2 + 5 + 1 =$

(9)
20
10
2
30
25
—

(10)
21
33
35
—

(11)
32
59
17
—

(12)
43
1
2
13
—

(13)
23
25
16
—

(14) $25 + 42 =$

(15)
100
33
45
201
46
—

(16)
9
24
12
15
19
—

(17)
199
194
295
156
—

(18)
2563
1387
4954
2065
—

(19)
$ .75
1.25
.49
—

(20)
$12.50
16.75
15.75
—

(21)
$8.00
5.75
2.33
4.16
.94
6.32
—

(22)
547
197
685
678
456
393
525
240
152
—

(23) $\frac{1}{3} + \frac{1}{3} =$

(24)
4.0125
1.5907
4.10
8.673
—

(25) $\frac{3}{8} + \frac{5}{8} + \frac{7}{8} + \frac{1}{8} =$

(26)
$12\frac{1}{2}$
$62\frac{1}{2}$
$12\frac{1}{2}$
$37\frac{1}{2}$
—

(27) $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} =$

(28) $\frac{3}{4} + \frac{1}{4} =$

(29)
$4\frac{3}{4}$
$2\frac{1}{4}$
$5\frac{1}{4}$
—

(30)
$2\frac{1}{2}$
$6\frac{3}{8}$
$3\frac{3}{4}$
—

(31)
113.46
49.6097
19.9
9.87
.0086
18.253
6.04
—

(32) $\frac{3}{4} + \frac{1}{2} + \frac{1}{4} =$

(33)
.49
.28
.63
.95
1.69
.22
.33
.36
1.01
.56
.88
.75
.56
1.10
.18
.56
—

(34) $\frac{1}{6} + \frac{3}{8} =$

(35)
2 ft. 6 in.
3 ft. 5 in.
4 ft. 9 in.
—

(36)
2 yr. 5 mo.
3 yr. 6 mo.
4 yr. 9 mo.
5 yr. 2 mo.
6 yr. 7 mo.
—

(37)
$16\frac{1}{3}$
$12\frac{1}{8}$
$21\frac{1}{2}$
$32\frac{3}{4}$
—

(38) $25.091 + 100.4 + 25 + 98.28 + 19.3614 =$

Series A

Subtraction Scale

By Clifford Woody

City..... County..... School..... Date.....
Name.....When is your next birthday?.....
How old will you be?.....Are you a boy or girl?.....
In what grade are you?.....Teacher's name.....

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
8	6	2	9	4	11	13	59	78	7 — 4 =	76
5	0	1	3	4	7	8	12	37		60

(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
27	16	50	21	270	393	1000	567482	$2\frac{3}{4} - 1 =$
3	9	25	9	190	178	537	106493	

(21)	(22)	(23)	(24)	(25)	(26)
10.00	$3\frac{1}{2} - \frac{1}{2} =$	80836465	$8\frac{7}{8}$	27	4 yds. 1 ft. 6 in.
3.49		49178036	$5\frac{3}{4}$	$12\frac{5}{8}$	2 yds. 2 ft. 3 in.

(27)	(28)	(29)	(30)
5 yds. 1 ft. 4 in.	10 — 6.25 =	$75\frac{3}{4}$	9.8063 — 9.019 =
2 yds. 2 ft. 8 in.		$52\frac{1}{4}$	

(31)	(32)	(33)	(34)	(35)
7.3 — 3.00081 =	1912 6 mo. 8 da.	$\frac{5}{12} - \frac{2}{10} =$	$6\frac{1}{8}$	$3\frac{7}{8} - 1\frac{5}{8} =$
	1910 7 mo. 15 da.		$2\frac{7}{8}$	

Series A

Multiplication Scale

By Clifford Woody

City..... County..... School..... Date.....
Name.....When is your next birthday?.....
How old will you be?.....Are you a boy or girl?.....
In what grade are you?.....Teacher's name.....

(1) $3 \times 7 =$

(2) $5 \times 1 =$

(3) $2 \times 3 =$

(4) $4 \times 8 =$

(5) 23 × 3 =

(6) 310 × 4 =

(7) $7 \times 9 =$

(8) 50 × 3 =

(9) 254 × 6 =

(10) 623 × 7 =

(11) 1036 × 8 =

(12) 5096 × 6 =

(13) 8754 × 8 =

(14) 165 × 40 =

(15) 235 × 23 =

(16) 7898 × 9 =

(17) 145 × 206 =

(18) 24 × 234 =

(19) 9.6 × 4 =

(20) 287 × .05 =

(21) 24 × $2\frac{1}{2}$ =

(22) $8 \times 5\frac{3}{4} =$

(23) $1\frac{1}{4} \times 8 =$

(24) $16 \times 2\frac{5}{8} =$

(25) $\frac{7}{8} \times \frac{3}{4} =$

(26) 9742 × 59 =

(27) 6.25 × 3.2 =

(28) .0123 × 9.8 =

(29) $\frac{1}{8} \times 2 =$

(30) 2.49 × .36 =

(31) $\frac{12}{25} \times \frac{15}{32} =$

(32) 6 dollars 49 cents × 8 =

(33) $2\frac{1}{2} \times 3\frac{1}{2} =$

(34) $\frac{1}{2} \times \frac{1}{2} =$

(35) $987\frac{3}{4} \times 25 =$

(36) 3 ft. 5 in. × 5 =

(37) $2\frac{1}{4} \times 4\frac{1}{2} \times 1\frac{1}{2} =$

(38) $.0963\frac{1}{8} \times .084 =$

(39) 8 ft. $9\frac{1}{2}$ in. × 9 =

Series A

Division Scale

By Clifford Woody

City..... County..... School..... Date.....
Name.....When is your next birthday?.....
How old will you be?.....Are you a boy or girl?.....
In what grade are you?.....Teacher's name.....

(1) $3\overline{)6}$ (2) $9\overline{)27}$ (3) $4\overline{)28}$ (4) $1\overline{)5}$ (5) $9\overline{)36}$ (6) $3\overline{)39}$

(7) $4 \div 2 =$ (8) $9\overline{)0}$ (9) $1\overline{)1}$ (10) $6 \times \ldots . = 30$ (11) $2\overline{)13}$ (12) $2 \div 2 =$

(13) $4\overline{)24 \text{ lbs. } 8 \text{ oz.}}$ (14) $8\overline{)5856}$ (15) $\frac{1}{4}$ of $128 =$ (16) $68\overline{)2108}$ (17) $50 \div 7 =$

(18) $13\overline{)65065}$ (19) $248 \div 7 =$ (20) $2.1\overline{)25.2}$ (21) $25\overline{)9750}$ (22) $2\overline{)13.50}$

(23) $23\overline{)469}$ (24) $75\overline{)2250300}$ (25) $2400\overline{)504000}$ (26) $12\overline{)2.76}$

(27) $\frac{7}{8}$ of $624 =$ (28) $.003\overline{).0936}$ (29) $3\frac{1}{2} \div 9 =$ (30) $\frac{3}{4} \div 5 =$

(31) $\frac{5}{4} \div \frac{3}{5} =$ (32) $9\frac{5}{8} \div 3\frac{3}{4} =$ (33) $52\overline{)3756}$

(34) $62.50 \div 1\frac{1}{4} =$ (35) $531\overline{)37722}$ (36) $9\overline{)69 \text{ lbs. } 9 \text{ oz.}}$

The addition scale begins with $2 + 3$ and includes the addition of increasingly difficult exercises. It brings in fractions, both with common denominators and with different denominators, mixed numbers, decimals, and compound numbers of two denominations.

The subtraction scale is made up of problems involving numbers of the same kind as those in the addition scale.

The multiplication scale includes the simple combinations, multiplication of integers by integers up to four figures in the multiplicand and two in the multiplier, a fraction by a fraction, a decimal by a decimal, and a compound number by an integer.

The division scale includes the simple combinations, short division, long division up to the division of a number of from five to seven digits by one of two or three digits, division of mixed numbers, fractions, decimals and compound numbers.

In giving these tests the time allowed is practically unlimited; twenty minutes being allowed for each test. In this length of time nearly all the pupils will have completed all the problems that they can solve. These tests

are then "power" tests rather than "speed" tests such as those devised by Courtis.

Another way in which the Woody scales differ from the Courtis tests is that in the latter the problems in a given test are of equal difficulty, while in the former they are of varying degrees of difficulty. This being the case it became necessary for Mr. Woody to adopt some unit by means of which the degree of difficulty of each problem could be stated. The unit adopted was the Probable Error (P. E.) of the school grade distribution. The median achievement of a grade distribution, i. e., a problem that is solved by exactly 50% of the grade, is taken as the measure of the achievement of the grade. The P. E. of a grade distribution is that distance along the base line of a surface distribution from the median point to the perpendicular on either side of the median which cuts off twenty-five per cent of the cases. The P. E. of the grade's distribution is the limits of the middle 50% of the grade. In other words if exactly 50% of a class are able to solve a problem correctly, then 25% of that class should be able to solve a problem that is at least one unit (P. E.) more difficult, and 75% of that class should be able to solve a problem one unit less difficult.

THE MONROE TESTS

The third series of tests included in this study is the one devised by Walter S. Monroe.

This author starts out deliberately to construct a series of tests of the operations in arithmetic that will include all or nearly all of the types of examples encountered in arithmetical work. He points out the fact that existing studies show that there are as many arithmetical abilities as there are types of examples and argues that any test that is to be really diagnostic must include all the important types within its scope.

According to Mr. Courtis there are six types of operations in the addition of integers, four in subtraction, nine in multiplication, and ten in division. Kallom has analyzed the addition of two fractions and reached the conclusion that there are fourteen types of examples. Mr. Monroe, without making a very careful analysis, carries the discussion of types on through fractions and decimals and reaches the conclusion that there are at least 86 significant types of examples in the fundamental operations of arithmetic (integers, 30; common fractions, 36; decimal fractions, 20 to 40). This is exclusive of those involved in the writing and reading of numbers, in the tables of denominate numbers, and in the solution of problems.

The 21 tests devised by Mr. Monroe contain 61 of these types. These tests are given in limited lengths of time so that they measure both speed and accuracy. In this respect they differ from the Woody tests and agree with the Cleveland tests. In fact Mr. Monroe argues that arithmetical abilities are "two dimensional," and that any attempt to measure them must take this fact into consideration. He admits, however, that the usual class-room procedure is to measure power only without much regard to speed.

The 21 tests are given here in full. They are printed in four different folders. The first two, containing tests 1-11, deal with the four fundamental operations with integers; the third, tests 12-16, deals with common fractions; the fourth, tests 17-21, with decimal fractions. The fourth folder was not used in this study.

Part I—Tests 1-6.

Bureau of Educational Measurements and Standards
Kansas State Normal School, Emporia, Kansas

DIAGNOSTIC TESTS IN ARITHMETIC
Operations With Integers
Devised by Walter S. Monroe

Name..... Age today.....
Years Months
City..... Grade..... Room.....
School..... Teacher..... Date today.....

Instructions to Examiners

Have the pupils fill out the blanks at the top of this page. Have them start and stop work together. Use a stop watch if one is available; if not, use an ordinary watch with a second hand and exercise care to allow just the exact time for each test. Allow an interval of half a minute or more between tests. Require the pupils to close the folder as soon as the signal to stop is given, in order to make certain that they do not spend this rest period working on the next test. If the pupils need to sharpen pencils before going on, allow this to be done. The following time allowances must be followed exactly:

Test 1—30 seconds.
Test 2—30 seconds.
Test 3—1 minute.
Test 4—1 minute.
Test 5—3 minutes.
Test 6—2 minutes.

Have the children read the following directions: "Inside this folder are examples which you are to work out when the teacher tells you to begin. Do not open this folder before the teacher gives the signal. Work rapidly and accurately. There are more examples in each test than you can work out in the time that will be allowed. Answers do not count if they are wrong. Begin and stop promptly at signals from the teacher. Place the test in position on your desk so that you can open it quickly when the signal is given to begin, but do not open it until the signal is given."

After all of the tests have been completed have the pupils exchange papers. Read the answers aloud and have the children mark each example that is correct "C." Count the number of examples attempted and the number of "C's" and write the numbers in the proper spaces at the top of the tests. Examples partially completed or partially right are not counted.

Before collecting the papers have the records transcribed to the first page. The teacher should verify a sufficient number of records to make certain that the pupils have marked the papers and transcribed the results correctly.

Test	1	2	3	4	5	6
Number of examples attempted						
Number of examples right						

At.....

Test 1—ADDITION.

Rt.....

4	5	2	0	1	7	6	7	3	2	3	9
7	5	6	3	1	2	8	7	8	4	3	4
2	9	7	8	4	3	4	0	9	0	6	5
—	—	—	—	—	—	—	—	—	—	—	—
8	8	5	4	4	1	0	0	7	6	6	3
0	9	9	6	5	5	2	1	1	8	7	7
5	2	1	1	8	7	7	4	3	3	0	9
—	—	—	—	—	—	—	—	—	—	—	—

At.....

Test 2—SUBTRACTION.

Rt.....

37	94	60	27	39	41	77	53
5	8	3	6	7	8	3	9
—	—	—	—	—	—	—	—
65	80	92	70	68	58	26	43
2	4	5	3	2	9	9	8
—	—	—	—	—	—	—	—
95	50	36	34	44	25	63	57
4	7	1	8	6	3	7	9
—	—	—	—	—	—	—	—

At.....

Test 3—MULTIPLICATION.

Rt.....

6572	6750	5863	3754	2845
6	9	2	5	8
4936	9327	8274	8409	6391
4	7	3	6	9
5482	8609	3679	2758	4658
2	5	8	4	7
9653	3174	2874	7901	2179
3	6	9	2	5

At.....

Test 4—DIVISION.

Rt.....

8)3840	4)7432	7)2534	3)8430	6)4680
9)8577	2)6370	5)9310	8)7512	4)3820
7)9653	3)5781	6)6720	9)5373	2)5130

Test 5—ADDITION.

At.....

Rt.....

7862	6809	8941	5917	6772	7864	1249
5013	7623	7910	4814	6028	7883	8975
1761	5299	9845	9007	6535	8240	9005
5872	6601	8522	6975	2340	9869	1573
3739	3496	1046	1227	2319	6794	3203
8758	2462	1247	4319	6794	3283	7917
2350	9869	3573	2358	5420	7805	4304
3197	4572	1081	5795	4570	7642	9027
2338	6420	7805	4314	8028	7803	9975
5917	6772	9864	1249	8758	2462	1247

Test 6—DIVISION.

At.....

Rt.....

82)3854	43)1591	74)2664	31)1953
63)3591	94)4042	21)1344	53)4452
83)5312	42)672	71)5183	32)2304
62)2108	93)5022	23)782	51)2703
84)7140	41)3567	73)6278	33)1386
64)5312	92)6624	24)984	52)3484

Bureau of Educational Measurements and Standards
Kansas State Normal School, Emporia, Kansas

DIAGNOSTIC TESTS IN ARITHMETIC

Operations With Integers

Devised by Walter S. Monroe

Name..... Age today.....
Years Months
City..... Grade..... Room.....
School..... Teacher..... Date today.....

Instructions to Examiners

Have the pupils fill out the blanks at the top of this page. Have them start and stop work together. Use a stop watch if one is available; if not, use an ordinary watch with a second hand and exercise care to allow just the exact time for each test. Allow an interval of half a minute or more between tests. Require the pupils to close the folder as soon as the signal to stop is given, in order to make certain that they do not spend this rest period working on the next test. If the pupils need to sharpen pencils before going on, allow this to be done. The following time allowances must be followed exactly:

Test 7—2 minutes.
Test 8—3 minutes.
Test 9—1 minute.
Test 10—2 minutes.
Test 11—4 minutes.

Have the children read the following directions: "Inside this folder are examples which you are to work out when the teacher tells you to begin. Do not open this folder before the teacher gives the signal. Work rapidly and accurately. There are more examples in each test than you can work out in the time that will be allowed. Answers do not count if they are wrong. Begin and stop promptly at signals from the teacher. Place the test in position on your desk so that you can open it quickly when the signal is given to begin, but do not open it until the signal is given."

After all of the tests have been completed have the pupils exchange papers. Read the answers aloud and have the children mark each example that is correct "C." Count the number of examples attempted and the number of "C's" and write the numbers in the proper spaces at the top of the tests. Examples partially completed or partially right are not counted.

Before collecting the papers have the records transcribed to the first page. The teacher should verify a sufficient number of records to make certain that the pupils have marked the papers and transcribed the results correctly.

Test	7	8	9	10	11
Number of examples attempted..	.....	.....	.....	.....	.....
Number of examples right.....	.....	.....	.....	.....	.....

At.....

Test 7—ADDITION.

Rt.....

7	6	6	8	2	1	2	8	8	3	2	6	9	5	7
6	8	7	7	9	3	2	3	9	9	4	3	7	8	8
6	8	0	9	9	8	5	4	5	1	1	6	4	9	0
5	9	1	3	2	3	1	8	7	8	4	4	9	7	2
0	9	3	5	6	6	7	5	2	1	2	8	8	3	1
5	5	4	8	0	1	1	2	0	7	6	9	3	3	8
1	1	0	0	4	6	7	7	8	6	3	2	3	9	9
8	8	7	7	7	1	4	4	4	5	3	0	9	0	6
7	7	5	3	5	5	0	2	2	2	3	1	8	7	8
3	7	5	4	2	4	5	9	1	1	1	2	0	7	6
3	4	6	6	4	2	4	5	9	1	1	1	2	0	7
1	5	4	5	7	5	3	5	6	0	2	2	2	3	1
2	4	6	9	7	9	7	5	7	8	3	4	4	4	5

At.....

Test 8—MULTIPLICATION.

Rt.....

4857	5718	6942	4065
36	92	58	47
9625	6123	7486	9027
23	64	75	89
1253	5376	3786	5492
38	76	49	53

At.....

Test 9—SUBTRACTION.

Rt.....

739	1852	975	1087	516	962
367	948	906	821	239	325
508	1371	1284	730	1853	897
447	843	966	508	162	258
1910	735	1056	877	1190	619
361	478	591	618	739	257
831	954	1077	1328	939	1316
360	483	704	872	654	827

At.....

Test 10—MULTIPLICATION.

Rt.....

560	807	617	840	730	609
37	59	508	80	96	70
435	790	940	307	682	870
308	60	38	42	409	40
780	502	386	150	850	401
56	68	207	90	72	80
817	460	730	605	392	590
109	30	52	84	306	30

At.....

Test 11—DIVISION.

Rt.....

47)27589	79)36893	36)28296	68)31824
96)56064	28)21980	57)22572	89)25365
48)32304	76)36708	67)39932	98)46844

Bureau of Educational Measurements and Standards
Kansas State Normal School, Emporia, Kansas

DIAGNOSTIC TESTS IN ARITHMETIC

Operations With Common Fractions

Devised by Walter S. Monroe

Name..... Age today..... Years Months
City..... Grade..... Room.....
School..... Teacher..... Date today.....

Instructions to Examiners

Have the pupils fill out the blanks at the top of this page. Have them start and stop work together. Use a stop watch if one is available; if not, use an ordinary watch with a second hand and exercise care to allow just the exact time for each test. Allow an interval of half a minute or more between tests. Require the pupils to close the folder as soon as the signal to stop is given, in order to make certain that they do not spend this rest period working on the next test. If the pupils need to sharpen pencils before going on, allow this to be done. The following time allowances must be followed exactly:

Test 12—1½ minutes.
Test 13—2 minutes.
Test 14—1 minute.
Test 15—2 minutes.
Test 16—2 minutes.

Have the children read the following directions: "Inside this folder are examples which you are to work out when the teacher tells you to begin. Do not open this folder before the teacher gives the signal. Work rapidly and accurately. There are more examples in each test than you can work out in the time that will be allowed. Answers do not count if they are wrong. Begin and stop promptly at signals from the teacher. Place the test in position on your desk so that you can open it quickly when the signal is given to begin, but do not open it until the signal is given."

After all of the tests have been completed have the pupils exchange papers. Read the answers aloud and have the children mark each example that is correct "C." Count the number of examples attempted and the number of "C's" and write the numbers in the proper spaces at the top of the tests. Examples partially completed or partially right are not counted.

Before collecting the papers have the records transcribed to the first page. The teacher should verify a sufficient number of records to make certain that the pupils have marked the papers and transcribed the results correctly.

Test	12	13	14	15	16
Number of examples attempted					
Number of examples right					

Test 12.—ADDITION.

At.....
Rt.....

Reduce your answers to lowest terms.

$\frac{1}{6} + \frac{1}{3} =$ $\frac{3}{10} + \frac{2}{5} =$ $\frac{5}{9} + \frac{2}{3} =$

$\frac{5}{6} + \frac{1}{2} =$ $\frac{1}{8} + \frac{1}{2} =$ $\frac{5}{6} + \frac{7}{12} =$

$\frac{3}{4} + \frac{1}{2} =$ $\frac{1}{3} + \frac{1}{12} =$ $\frac{1}{2} + \frac{7}{10} =$

$\frac{3}{4} + \frac{5}{12} =$ $\frac{5}{8} + \frac{1}{4} =$ $\frac{1}{2} + \frac{5}{12} =$

$\frac{1}{6} + \frac{2}{3} =$ $\frac{4}{5} + \frac{7}{10} =$ $\frac{5}{8} + \frac{3}{4} =$

Test 13—SUBTRACTION.

At.....
Rt.....

Reduce your answers to lowest terms.

$\frac{3}{4} - \frac{2}{5} =$ $\frac{5}{6} - \frac{3}{4} =$ $\frac{1}{2} - \frac{2}{7} =$

$\frac{7}{10} - \frac{1}{6} =$ $\frac{2}{3} - \frac{1}{2} =$ $\frac{5}{6} - \frac{2}{15} =$

$\frac{3}{4} - \frac{1}{3} =$ $\frac{7}{9} - \frac{1}{6} =$ $\frac{2}{3} - \frac{3}{5} =$

$\frac{5}{6} - \frac{3}{8} =$ $\frac{3}{4} - \frac{2}{7} =$ $\frac{7}{12} - \frac{3}{8} =$

$\frac{5}{6} - \frac{3}{5} =$ $\frac{8}{15} - \frac{4}{9} =$ $\frac{4}{5} - \frac{1}{3} =$

Test 14—MULTIPLICATION.

At.....
Rt.....

Reduce your answers to lowest terms.

$\frac{2}{3} \times \frac{3}{4} =$ $\frac{2}{5} \times \frac{3}{7} =$ $\frac{5}{12} \times \frac{3}{5} =$

$\frac{4}{9} \times \frac{2}{5} =$ $\frac{1}{3} \times \frac{3}{8} =$ $\frac{1}{2} \times \frac{1}{3} =$

$\frac{2}{5} \times \frac{3}{4} =$ $\frac{4}{5} \times \frac{1}{3} =$ $\frac{7}{12} \times \frac{4}{7} =$

$\frac{3}{8} \times \frac{1}{4} =$ $\frac{2}{7} \times \frac{1}{6} =$ $\frac{1}{3} \times \frac{1}{2} =$

$\frac{4}{15} \times \frac{5}{8} =$ $\frac{4}{5} \times \frac{7}{9} =$ $\frac{1}{6} \times \frac{3}{10} =$

At.....

Rt.....

Test 15—ADDITION

Reduce your answers to lowest terms.

$\frac{1}{6} + \frac{3}{5} =$ $\frac{3}{12} + \frac{5}{8} =$ $\frac{3}{5} + \frac{1}{2} =$

$\frac{4}{9} + \frac{1}{6} =$ $\frac{1}{3} + \frac{4}{7} =$ $\frac{4}{15} + \frac{5}{9} =$

$\frac{1}{2} + \frac{2}{3} =$ $\frac{7}{10} + \frac{3}{8} =$ $\frac{1}{3} + \frac{3}{4} =$

$\frac{3}{8} + \frac{5}{6} =$ $\frac{1}{7} + \frac{2}{5} =$ $\frac{1}{10} + \frac{1}{15} =$

$\frac{2}{5} + \frac{2}{3} =$ $\frac{3}{10} + \frac{1}{4} =$ $\frac{4}{7} + \frac{3}{5} =$

At.....

Rt.....

Test 16—DIVISION.

Reduce your answers to lowest terms.

$\frac{2}{5} \div \frac{1}{3} =$ $\frac{4}{7} \div \frac{2}{3} =$ $\frac{3}{8} \div \frac{2}{3} =$

$\frac{5}{6} \div \frac{5}{8} =$ $\frac{3}{7} \div \frac{4}{5} =$ $\frac{7}{12} \div \frac{4}{9} =$

$\frac{1}{2} \div \frac{1}{3} =$ $\frac{2}{3} \div \frac{8}{9} =$ $\frac{2}{3} \div \frac{3}{4} =$

$\frac{4}{7} \div \frac{8}{11} =$ $\frac{3}{5} \div \frac{3}{4} =$ $\frac{1}{4} \div \frac{1}{6} =$

$\frac{4}{5} \div \frac{1}{2} =$ $\frac{2}{5} \div \frac{3}{7} =$ $\frac{5}{12} \div \frac{4}{9} =$

Part IV. Tests 17-21.

Bureau of Educational Measurements and Standards
Kansas State Normal School, Emporia, Kansas

DIAGNOSTIC TESTS IN ARITHMETIC

Multiplication and Division of Decimal Fractions

Devised by Walter S. Monroe

Name..... Age today.....
Years Months
City..... Grade..... Room.....
School..... Teacher..... Date today.....

Instructions to Examiners

Have the pupils fill out the blanks at the top of this page. Have them start and stop work together. Use a stop watch if one is available; if not, use an ordinary watch with a second hand and exercise care to allow just the exact time for each test. Allow an interval of half a minute or more between tests. Require the pupils to close the folder as soon as the signal to stop is given, in order to make certain that they do not spend this rest period working on the next test. If the pupils need to sharpen pencils before going on, allow this to be done. The following time allowances must be followed exactly:

Test 17—30 seconds.
Test 18—30 seconds.
Test 19—30 seconds.
Test 20—30 seconds.
Test 21—30 seconds.

Have the children read the following directions: "Inside this folder are examples which you are to work out when the teacher tells you to begin. Do not open this folder before the teacher gives the signal. Work rapidly and accurately. There are more examples in each test than you can work out in the time that will be allowed. Answers do not count if they are wrong. Begin and stop promptly at signals from the teacher. Place the test in position on your desk so that you can open it quickly when the signal is given to begin, but do not open it until the signal is given."

After all of the tests have been completed have the pupils exchange papers. Read the answers aloud and have the children mark each example that is correct "C." Count the number of examples attempted and the number of "C's" and write the numbers in the proper spaces at the top of the tests. Examples partially completed or partially right are not counted.

Before collecting the papers have the records transcribed to the first page. The teacher should verify a sufficient number of records to make certain that the pupils have marked the papers and transcribed the results correctly.

Test	17	18	19	20	21
Number of examples attempted					
Number of examples right					

At.....

Test 17—DIVISION.

Rt.....

The correct answer for each example with the exception of the decimal point is given at the side immediately after the letters "Ans." Write the answer in its proper position and place the decimal point in its proper place. Place ciphers before or after the answer when they are necessary.

.03)16.2 Ans.: 54	.07)1.82 Ans.: 26	.05).415 Ans.: 83
.06)7.44 Ans.: 124	.08).952 Ans.: 119	.04)87.6 Ans.: 219
.02).144 Ans.: 72	.08)40.8 Ans.: 51	.09)3.42 Ans.: 38
.03)47.4 Ans.: 158	.07)8.61 Ans.: 123	.05).965 Ans.: 193
.09)5.76 Ans.: 64	.04).348 Ans.: 87	.06)51.0 Ans.: 85
.02).748 Ans.: 374	.03)89.1 Ans.: 297	.05)6.85 Ans.: 137
.09)94.5 Ans.: 105	.01)5.48 Ans.: 548	.06).288 Ans.: 48
.04)9.84 Ans.: 246	.07).238 Ans.: 34	.08)44.8 Ans.: 56

At.....

Test 18—MULTIPLICATION.

Rt.....

Place the decimal point correctly in the following products:

657.2	67.50	5.863	375.4	28.45	4.936
.7	.03	.6	.09	.2	.05
46004	20250	35178	33786	5690	24680
932.7	82.74	8.409	639.7	54.82	8.609
.08	.4	.07	.3	.06	.9
74616	33096	58863	19191	32892	77481
367.9	27.58	4.658	965.3	31.74	2.874
.2	.05	.8	.04	.7	.03
7358	13790	37264	38612	22218	8622
574.6	82.47	7.462	834.7	54.32	7.842
.06	.9	.02	.5	.08	.4
34476	74223	14924	41735	43456	31368

At.....

Test 19—DIVISION.

Rt.....

The correct answer for each example with the exception of the decimal point is given at the side immediately after the letters "Ans." Write the answer in its proper position and place the decimal point in its proper place. Place ciphers before or after the answer when they are necessary.

.4)148. Ans.: 37	.9)65.7 Ans.: 73	.6)1.68 Ans.: 28
.7).301 Ans.: 43	.3)47.7 Ans.: 159	.6)8.34 Ans.: 139
.2).548 Ans.: 274	.4)744. Ans.: 186	.3)117. Ans.: 39
.9).756 Ans.: 74	.8)672. Ans.: 84	.7)59.5 Ans.: 85
5) 865 Ans.: 173	.3)684. Ans.: 228	.6)93.6 Ans.: 156
.2)7.92 Ans.: 396	.4)352. Ans.: 88	.3)16.2 Ans.: 54
.7)3.22 Ans.: 46	.5).710 Ans.: 142	.8)376. Ans.: 47
.1)9.42 Ans.: 942	.6).852 Ans.: 142	.2)74.2 Ans.: 371

At.....

Test 20—MULTIPLICATION.

Rt.....

Place the decimal point correctly in the following products:

487.5 .62	57.28 9.5	6.294 .28	4065. 5.1	967.5 8.4
302250	544160	176232	207315	712700
61.32 .17	7.465 4.3	7486. .76	907.2 .39	14.53 6.2
104244	320995	558936	353808	90086
5.376 .91	8637. 2.4	549.3 5.7	84.74 .83	8.637 1.6
489216	207588	313101	703342	138192
5194. .49	784.1 .72	36.74 3.5	2.893 .68	4936. 9.4
254506	564552	128590	196724	463984

At.....

Test 21—DIVISION.

Rt.....

The correct answer for each example, with the exception of the decimal point, is given below the quotient, after the letters, "Ans." Write the answer in its proper position and place the decimal point in its proper place. Place ciphers before or after the answer when necessary.

.47)2758.9	8.2)38.54	79.)36.893	.43)1591
Ans.: 587	Ans.: 47	Ans.: 467	Ans.: 37
3.6)2829.6	74.)26.64	.68)31.824	3.1)1953.
Ans.: 786	Ans.: 36	Ans.: 468	Ans.: 63
96.)5606.4	.63)35.91	2.8)21.980	94.)4.042
Ans.: 584	Ans.: 57	Ans.: 785	Ans.: 43
.57)22572.	2.1)140.7	89.)253.65	.53)4.452
Ans.: 396	Ans.: 67	Ans.: 285	Ans.: 84
4.8)32304.	83.)531.2	.76)367.08	4.2).672
Ans.: 673	Ans.: 64	Ans.: 483	Ans.: 16

PART II

Having in mind the purpose and character of the tests to be used we may now turn to the main question at issue in the study, viz., do the different tests agree as to results? If they do the fact may be taken as a strong indication that they are all well suited to their purpose. If they disagree then certainly one or more of the tests is faulty in some respect or else they do not measure the same abilities.

The tests were given on six successive school days, beginning October 23, to a group of about 60 eighth grade pupils in Manhattan, Kansas. The order followed was Cleveland tests, Monroe tests, and Woody scales.

The tests were all given and the scores checked by the author. Care was exercised to see that conditions were as nearly identical in the different tests as it was possible to make them.

The results of the tests are shown in Tables 1 to 6, and diagrams 1 to 6.

Table 1 shows a comparison of the standard scores and the class scores for the number of problems solved correctly and the per cent of accuracy in each of the Cleveland tests. The standards shown here are the averages of the Cleveland, Grand Rapids and St. Louis median scores in the 8B sections. Table 2 gives the standard scores and class scores in attempts and in per cent of accuracy for the Monroe tests. In both of these tables the tests are arranged in such order as to bring together all the tests in each of the four fundamental operations. Tables 3, 4, 5 and 6 show the results of the Woody tests.

These results are shown in graphic form in diagrams 1, 2, 3 and 4. In these diagrams the horizontal lines represent the grades, the vertical lines the tests and the figures at the points of intersection the standard scores of the different grades in the indicated tests. The broken line represents the class scores as determined by this series of tests.

Comparison of Standard and Class Scores

Table 1—Cleveland Survey Tests

	Standard Scores		Class Scores	
Test	Rts.	Ac.	Rts.	Ac.
A	29.8	99	24.5	99
E	7.8	94	5.2	93
J	5.6	78	3.7	70
M	5.3	76	4.6	87
B	25.2	99	18.2	95
F	10.2	90	7.1	83
C	19.7	89	16.3	87
G	6.9	88	5.5	90
L	4.7	69	3.6	69
D	22.3	97	18.7	98
I	4.7	84	2.8	70
K	10.8	95	7.6	94
N	2.4	81	1.5	68
H	9.3	77	5.6	89
O	5.7	68	3.5	47

Table 2—Monroe Tests.

	Standard Scores		Class Scores	
Test	Ats.	Ac.	Ats.	Ac.
1	12.7	100	12.5	100
7	5.4	79	4.9	81
5	6.1	66	5.4	62
2	8.9	100	7.9	100
9	8.5	97	8.1	100
3	6.2	84	5.6	86
8	6.5	73	6.1	81
10	6.6	82	4.9	90
4	4.6	88	4.9	100
6	4.5	100	3.4	100
11	3.4	68	3.0	100
12	9.8	73	7.8	76
15	8.5	59	6.5	76
13	7.8	71	6.8	81
14	13.5	75	9.6	93
16	8.5	59	9.7	82

Table 3—Woody Addition Scale

No. of Problem	No. Getting Each Prob.	% Getting Each Prob.
1	58	98
2	59	100
3	59	100
4	59	100
5	58	98
6	58	98
7	58	98
8	58	98
9	58	98
10	56	95
11	56	95
12	55	93
13	59	100
14	55	93
15	54	92
16	55	93
17	52	88
18	55	93
19	54	92
20	53	90
21	47	80
22	41	70
23	54	92
24	49	84
25	53	90
26	47	80
27	51	87
28	53	90
29	44	75
30	43	73
31	37	63
32	45	76
33	36	61
34	48	81
35	36	61
36	36	61
37	30	51
38	24	34

Standard Score, 9.01; Class Score, 8.76

Table 4—Woody Subtraction Scale

No. of Problem	No. Getting Each Prob.	% Getting Each Prob.
1	58	98
2	59	100
3	58	98
4	59	100
5	59	100
6	59	100
7	59	100
8	58	98
9	59	100
10	59	100
11	57	96
12	59	100
13	58	98
14	57	96
15	55	93
16	57	96
17	53	90
18	54	92
19	50	85
20	53	90
21	44	75
22	54	92
23	49	84
24	51	87
25	43	73
26	45	76
27	37	63
28	45	76
29	51	87
30	45	76
31	39	66
32	31	52
33	42	71
34	36	61
35	40	68

Standard Score, 7.64; Class Score, 7.99

Table 5—Woody Multiplication Scale

No. of Problem	No. Getting Each Prob.	% Getting Each Prob.
1	58	98
2	59	100
3	59	100
4	59	100
5	59	100
6	59	100
7	58	98
8	59	100
9	55	93
10	56	95
11	58	98
12	58	98
13	51	86
14	58	98
15	56	95
16	44	75
17	53	90
18	54	92
19	55	93
20	55	93
21	56	95
22	56	95
23	54	92
24	53	90
25	48	81
26	45	76
27	53	90
28	50	85
29	54	52
30	51	87
31	52	88
32	43	73
33	46	78
34	42	71
35	34	57
36	34	57
37	36	61
38	25	42
39	27	46

Standard Score, 7.93; Class Score, 8.19

Table 6—Woody Division Scale

No. of Problem	No. Getting Each Prob.	% Getting Each Prob.
1	55	96
2	57	100
3	57	100
4	57	100
5	57	100
6	57	100
7	57	100
8	55	96
9	55	96
10	57	100
11	55	96
12	55	96
13	51	89
14	57	96
15	52	91
16	51	89
17	52	91
18	41	68
19	51	89
20	48	84
21	53	93
22	49	86
23	36	63
24	43	75
25	42	74
26	44	77
27	49	86
28	45	79
29	34	60
30	36	63
31	39	68
32	43	75
33	36	63
34	33	58
35	22	39
36	8	14

Standard Score, 7.16; Class Score, 7.15

As a whole the group made the poorest showing in the Cleveland tests and the best in the Woody tests. This is undoubtedly due in part to the fact that the Cleveland tests were given first. It also indicates that the Cleveland standards are higher than either of the others. In the Cleveland tests the scores are all below standard; only one of them reached seventh grade standard, five are between seventh and sixth, seven between sixth and fifth, and two below fifth grade.

In the Monroe tests the score in one test is above standard, those in six tests are between seventh and eighth grade standards, and those in the three remaining tests are below sixth grade standards. On the Woody scale three are above standard and one between seventh and eighth grades.

There is then, even in this general statement, a serious discrepancy between the results obtained from the Woody scales and those obtained from the other two tests. Using the first named the teacher or supervisor would be led to the conclusion that these pupils did not need much more drill on the fundamentals. Using either of the others he would come to exactly the opposite conclusion.

But leaving the standards out of consideration let us see how the results agree as to the strength or weakness of the group tested in the different operations. Both the Cleveland and the Monroe tests show weakness in addition, the former to a greater extent than the latter, a lesser degree of weakness in subtraction and multiplication and irregularity in division and in fractions. The Woody tests agree with this showing in a general way, but they put subtraction considerably above any of the other operations.

Turning now to a study of the particular abilities in the various operations let us see what the different tests show. Test A, Cleveland, shows the group to be below sixth grade attainment in knowledge of addition combinations. The Monroe tests do not include problems of this character, but the Woody addition scale has two problems, Nos. 1 and 7. Neither of these shows any weakness here as both were solved correctly by all but one member of the group.

Test E, Cleveland, addition of 5 figure columns of single digits, indicates slightly better than fourth grade attainment, the weakest point in addition. Test 1, Monroe, 3 figure columns of single digits, shows between seventh and eighth grade attainment, the highest point in addition. Of course these tests are not identical in character and these results seem to indicate that they are not even of the same type. Problem 2, Woody addition scale, a column of three figures, was solved correctly by every member of the group, showing no weakness in this character of work.

Test J, Cleveland, addition of long columns requiring the bridging of the memory span, shows a score below fifth grade attainment, but slightly better than test E. Test 7, Monroe, gives a score below seventh grade standard, the weakest point in addition. The Woody scale does not give a problem of this character.

Test M, Cleveland, column addition four numbers wide and five deep, gives a score equal to seventh grade standard, the highest point in addition. Test 5, Monroe, of exactly the same character, also gives a score equal to seventh grade standard. Problem 18, Woody addition scale, was solved correctly by 93% of the class, a showing which agrees fairly well with the other two results.

Test B, Cleveland, subtraction combinations, shows a score a little below sixth grade. The Monroe tests do not include this type, but problems 1 to 7 and problem 10, Woody subtraction scale, show no weakness at all, being solved correctly by practically every member of the class.

Test F, Cleveland, subtraction involving borrowing, gives a score between fifth and sixth grade standards. Test 9, Monroe, gives a score between seventh and eighth grade standards. Problems 16, 17, 18, 19 and 23, Woody sub-

DIAGRAM 1

Median Scores in Rights, Cleveland Test

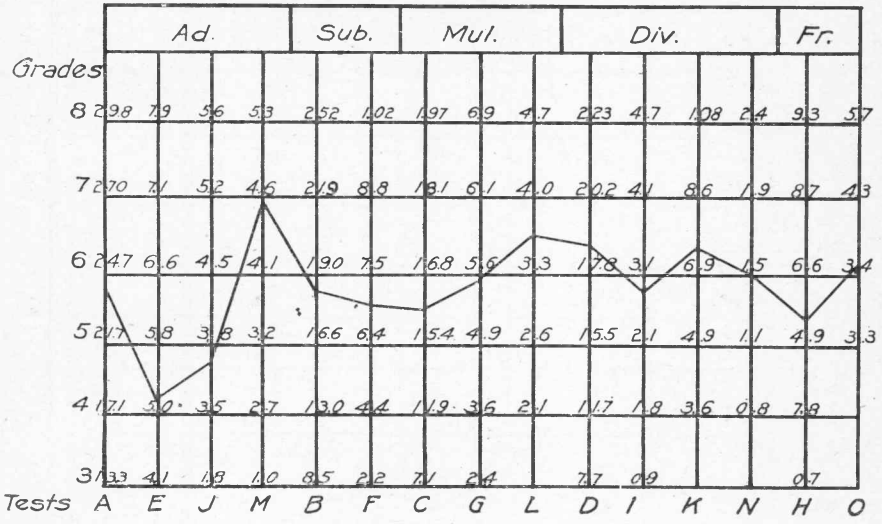


DIAGRAM 2

Median Scores in Attempts, Monroe Test

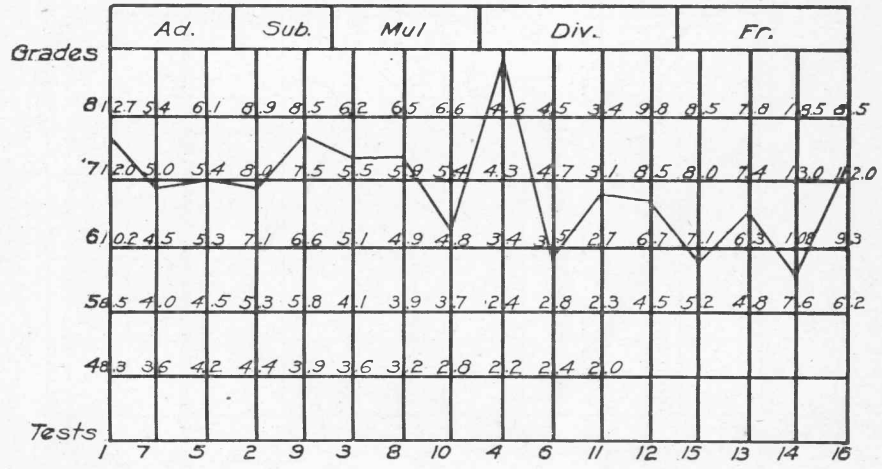


DIAGRAM 3

Class Scores, Woody Scales

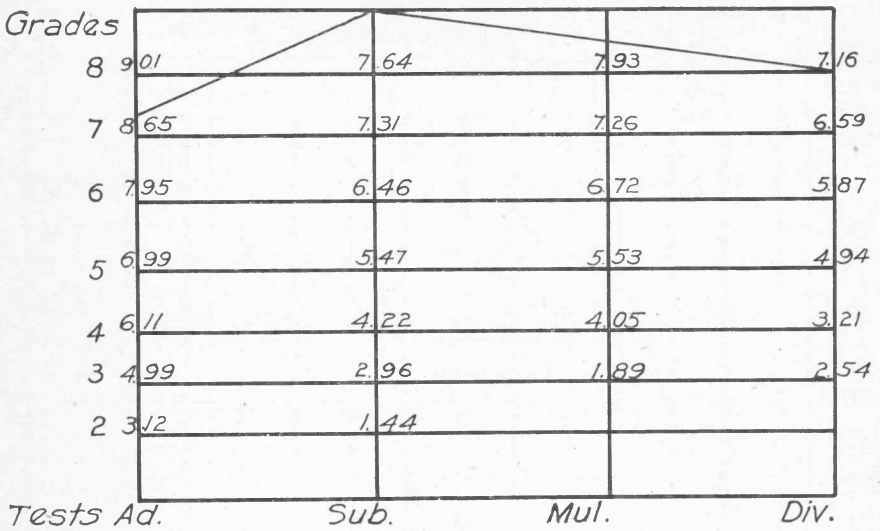


DIAGRAM 4

Cleveland and Monroe Medians

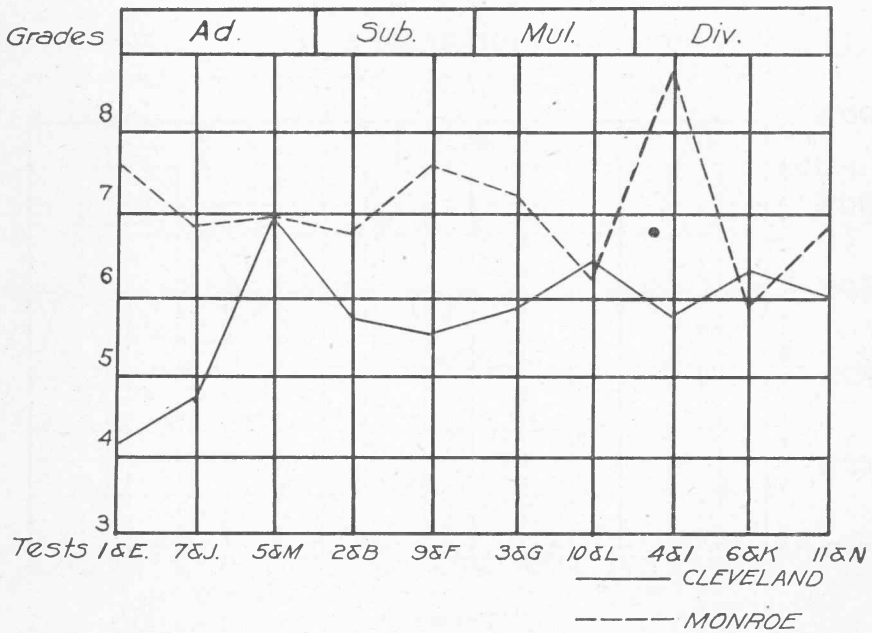


DIAGRAM 5

Accuracy Graphs, Cleveland Test

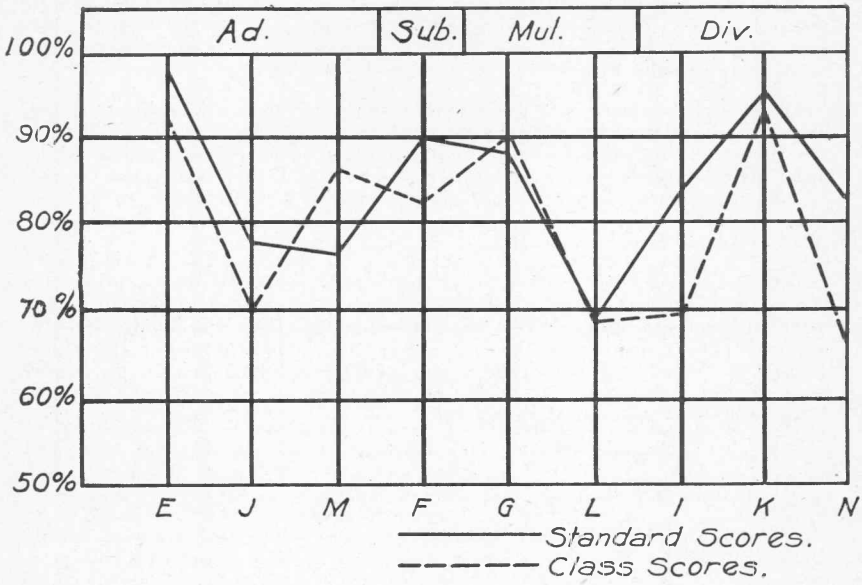
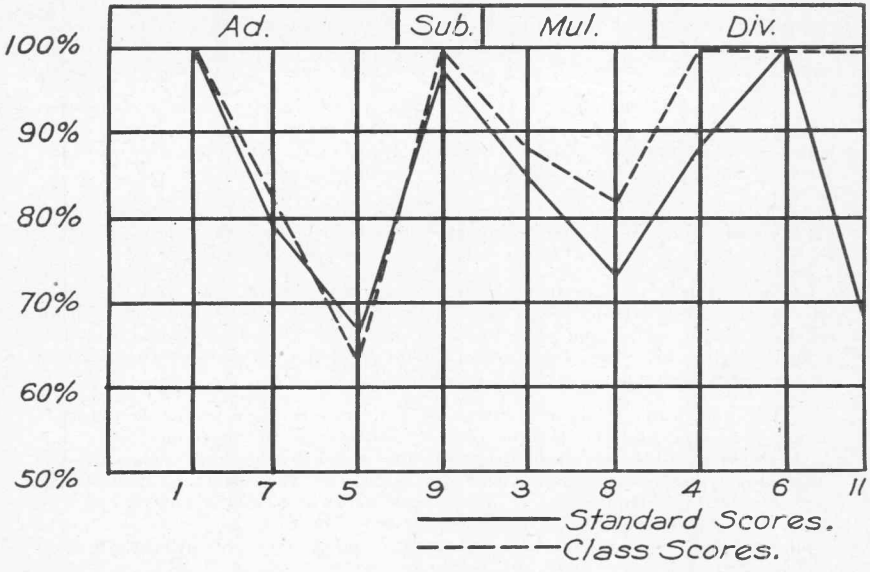


DIAGRAM 6

Accuracy Graphs, Monroe Test



traction scale, were solved correctly by 96, 90, 92, 85 and 84 per cent, respectively, or an average of 89 per cent, which indicates a weakness comparable to that shown by the Monroe test.

Test C, Cleveland, multiplication combination, places the children in this group below sixth grade standard. The Monroe tests do not include this type, and the Woody multiplication scale problems 1 to 4 and 7 again fail to show any weakness.

Test G, Cleveland, multiplication of numbers of 4 digits by a single digit, rates the group a little below sixth grade standard. Test 3, Monroe, the same kind of exercises, rates them a little below seventh grade, while problems 8, 9, 10, 11, 12, 13 and 16, Woody multiplication scale, show scores of 100, 93, 95, 98, 86 and 75 per cents, respectively. Of these 13 and 16 show decided weakness. These require multiplication by 8 and 9. Here again, then, the three tests are in substantial agreement.

Test L, Cleveland, multiplication by numbers of two digits, gives the highest score made in the Cleveland multiplication tests, midway between sixth and seventh grades. Test 8, Monroe, places the score above seventh grade standard, and is also the best score made in multiplication. Problems 15 and 26, Woody multiplication scale, give scores of 95 and 76 per cents, respectively, slightly better than the scores for the preceding type. The three tests here show substantial agreement.

Test D, Cleveland, division combinations, shows the class to be below seventh grade standard, while problems 1 to 5, 7, 8, 9 and 12, Woody division scale, give scores indicating practically no weakness at all.

Test I, Cleveland, short division, gives a score below sixth grade standard, while test 4, Monroe, gives a score above eighth grade standard. This is a discrepancy that is difficult to account for. It would seem to indicate that the small amount of practice the children received in short division in taking the Cleveland tests made a decided difference in their ability to perform this process when they took the second test. Problem 4, Woody division scale, gives a score of 96 per cent. Examination of the papers, however, shows that most of the children used the process of long division, so that the result gives no information concerning their ability in short division.

In test K, Cleveland, long division with small units digits in the divisor, the children scored a little above sixth grade standard. In test 6, Monroe, they scored a little below sixth grade. In problems 23 and 33, Woody division scale, they scored 63% and 63% respectively, a close agreement throughout.

Test N, Cleveland, long division, where the units digit in the divisor is large, shows sixth grade standard. Test 11, Monroe, shows a score a little below seventh grade standard. Problem 16, Woody division scale, shows a score of 89%. Here again we have substantial agreement.

The tests in fractions are not enough alike in type to make direct comparisons of value. In the main, however, all three of the tests show the class to be weak in their knowledge of the operations with fractions.

There is lack of agreement then between the Woody scale and the other two on the combinations in all four fundamental operations, column addition of three single digits, and short division, and substantial agreement in addition involving carrying, subtraction involving borrowing, multiplication by one or more digits, long division, and fractions.

Diagrams 5 and 6 show a comparison of the accuracy scores as obtained in the Cleveland and the Monroe tests in those types of problems that occur in both sets. Both of these graphs show a much closer approach to the standards than was found in either the rights for the Cleveland test or the attempts for the Monroe tests. They both indicate that the children do not vary from the standards so much in accuracy as they do in speed. In the main the two tests show rather close agreement as to results, the excep-

tions being in short column addition involving carrying, where the Cleveland test shows the higher degree of accuracy, short division, where the Monroe test gives the better showing, and long division with large units digits in the divisors, where the Monroe test again gives the better showing. The decided increase in accuracy in division shown by the Monroe tests over the Cleveland tests is probably due to the fact that by the time the children came to the Monroe tests in division they had discovered the fact that the division examples all come out without a remainder. This enabled them to detect errors and correct them.

The Woody scales do not give any adequate measure of accuracy.

This study shows then that there is a substantial agreement between the results obtained by using the Cleveland tests and those obtained by using the Monroe tests. The Monroe standards, however, seem to be distinctly lower than the median scores obtained by the use of the Cleveland tests in Cleveland, Grand Rapids and St. Louis. Considering the fact that this study was made in October while the Monroe standards are mid-year results it would seem that they are too low.

The Woody scale, on the other hand, gives results that differ materially from those obtained by the use of the other two tests. As has already been noted this scale places the class above standard in everything but addition and not far below standard even there, while both the others show them to be distinctly below standard in all the operations. Then it fails altogether to show weakness in the combinations and in the simple problems, a weakness clearly shown by both of the other tests. It fails also to show differences in the abilities in these simpler operations of the different children in a class. That marked differences do exist was clearly shown by the distributions on the score sheets for both the other tests. The reason for this failure is not far to seek. Even if a child does not know his combinations he can count up the results in the simpler problems and thus secure correct results if he has plenty of time, and the Woody scales give practically unlimited time, for most of the children finish each of these scales in less than the twenty minutes allowed.

The Woody scale would seem to be deficient then in several ways: (1) a test in fundamental operations should measure both speed and accuracy, as well as a knowledge of the process involved, (2) the number of problems of each type is too few to give an adequate measure of ability, (3) it fails to show individual differences between pupils or even classes in all of the simpler processes, (4) there is a lack of definiteness in the results obtained for the particular weakness (for instance, the results of the tests in this study show that the class is below standard in addition, but they do not tell us, except in a very indefinite way, how they compare with other eighth grade classes in column addition, in bridging the memory span, etc.), (5) its results are of little value in measuring individuals, while both the other tests can be used to great advantage in this regard. On the other hand the Woody test has some good points. It covers a wider field than either of the other tests. While it fails on the combinations and simple exercises, at least for upper grade work, it does show strength or weakness in the more important exercises, the ones that are most needed. It is in fact a test of neither speed nor accuracy, but rather a test of power. It can be used to advantage to determine which processes have been mastered by a class and which ones are still beyond them.

The Cleveland tests could be considerably improved by putting the four fundamental operations in fractions into four different tests instead of running them together in the two tests, H and O. The arrangement in test H is particularly bad. In all of the tests up to this point the pupils have had a single operation to perform, so that many of them when they come to test H and start in by adding the first two fractions, go right on and add all the others. So marked is this tendency that the results obtained from this test as it stands are practically worthless.

The Monroe tests could be greatly improved by printing the exercises in multiplication and long division so there would be more space for the work. As they are they make the work so crowded that the children are seriously hampered.

This study shows, then, that tests of the Cleveland Survey type are superior for the purpose of diagnosing strength or weakness in the operations of arithmetic and that those of the Woody type are decidedly inferior in this regard. They have their principal value in determining what processes have been mastered by any given class. Both types are valuable, but each should be used in the kind of diagnosis for which it is best fitted.