A STUDY IN ADDITION

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Professor of Educational Psychology

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RESEARCH BULLETIN
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Copies of this Bulletin may be obtained by writing to the State Teachers College of Colorado, Greeley, Colorado.

FOREWORD

This is the first of a series of Research Bulletins, which it is hoped, may be issued from The Colorado State Teachers College with a greater or less degree of regularity. That such bulletins may be of much benefit to the teaching profession there can be no question.

This bulletin is the result of weeks and months of the most careful and painstaking effort upon the part of the writers, and if it arrives at deductions and conclusions, striking and unexpected to teachers, it only proves that they should stop and take their educational bearings once in a while.

For example, it is almost certain that most teachers would say at once that the child knows that $9+1=10$ better than he knows that $9+9=18$; and yet the experiments recorded in this bulletin go to establish the reverse. Also the statement relative to memorizing addition combinations from tables or otherwise, rather than to learn them objectively or by roundabout methods, may appear unusual to many teachers.

This, the first bulletin of this series, is commended to the serious study of teachers as a real contribution to the pedagogy of arithmetic.

JAMES H. HAYS,
Acting President, State Teachers College.

Greeley, Colo., March 24, 1916.

A Study in Addition

The main purpose of making this study was to improve our knowledge of the best methods of teaching addition. The first specific problem which we undertook to investigate was the arrangement of the simple combinations in addition in a series which would represent the order of their difficulty. In connection with this problem there arose several others which are also treated in this article. The one was to determine the different methods of adding employed by children, the other was to study the value of drill.

The experimental part of our investigation was made in November, 1914. For the experiments which were to determine the order of difficulty, twenty pupils whose adding ability ranged from good to poor were selected from each of the fifth and sixth grades, the sexes being equally represented. We felt that the children in the lower grades did not know the combinations well enough for our purpose, and those in the higher grades knew them too well. Even in the fifth and sixth grades there were some children who knew all of the combinations about equally well and some who did not know many of the combinations at all. The records of such children were rejected and others were taken in their place.

The pupils were taken one at a time into a quiet room and each one was tested on the following table of addition combinations. The numbers to be added were presented orally, and the children responded orally with the sum. The experimenter, for example, called out 6, 7 with a short pause between the numbers and the child said 13 as soon as possible after having heard the numbers. The time was taken with a stop watch. The watch was started by the experimenter with the pronunciation of the last number and stopped as soon as the child's response was heard. The results were noted by an assistant. Those combinations which required the longest average response times were assumed to be the most difficult. If the association between two numbers and their sum is very intimate, the time to reproduce the sum in response to the numbers will in many trials be shorter than when the association is not very intimate or well known. Table I shows the combinations used and the order in which they were presented. The second division of the table shows the simple combinations which were not used in the experiment.

TABLE I.

Combinations used in the experiment.

4+9	7+6	7+5	5+5	7+4	9+5	9+1	7+2	2+2	3+4
5+8	8+5	8+4	8+9	8+7	6+5	8+2	8+3	4+3	2+4
7+7	9+4	4+7	4+5	9+6	9+8	9+2	6+1	5+3	3+1
8+6	4+8	5+6	6+9	4+4	6+4	7+3	5+2	6+3	4+1
5+9	5+7	7+8	7+9	9+9	8+8	8+1	7+1	3+3	1+3
6+8	6+6	4+6	5+4	6+7	9+7	9+3	1+1	4+2	

Combinations not used in the experiment.

6+2	3+2	3+6	3+8	2+1	2+5	2+7	2+9	1+4	1+6	1+8
5+1	3+5	3+7	3+9	2+3	2+6	2+8	1+2	1+5	1+7	1+9

To enhance the value of this part of our study, the experimental work on which it was based was repeated with a few modifications in February, 1916. This time all of the simple addition combinations were used and arranged for presentation as shown in Table II. The arrangement was made by chance with a few changes to avoid the immediate repetition of numbers and the direct recurrence of equal and many difficult sums. For half the boys and girls in a grade the numbers were presented in the order in which they appear in the table, but for the other half the order was reversed. The combinations which the children missed and those for which their response times were exceptionally long were presented again after the whole series had been gone through. In this way the errors were reduced to a negligible number, and the abnormally long reactions were eliminated.

TABLE II.

Combinations used in the second experiment.

5+2	3+2	9+5	9+9	3+3	8+1	2+7	8+5	6+9	9+4	9+7	1+7
3+9	4+9	7+8	5+6	4+7	3+5	7+5	9+3	4+4	7+7	2+2	2+5
7+2	6+8	2+6	7+1	2+8	8+4	6+1	1+6	6+7	5+4	6+4	5+7
1+9	2+3	9+2	4+8	4+2	1+1	8+9	5+8	2+4	2+1	4+3	1+8
5+3	7+9	3+1	5+9	3+6	9+6	3+7	1+4	6+3	8+2	1+5	
8+7	4+1	7+4	1+3	9+8	8+3	6+2	4+5	7+6	6+6	3+8	
5+1	8+8	8+6	2+9	6+5	4+6	3+4	9+1	1+2	7+3	5+5	

In Table III the combinations used in the first experiment are given in the order of their difficulty, the most difficult as determined by the response time appearing first. The table is divided into five groups. The first group contains all of the most difficult combinations which do not vary more than one second in their response times. The second group contains all of the next most difficult combinations which do not vary more than one-half second in response time, and the three remaining groups were made upon the same basis. The time which appears after each combination is expressed in fifths of a second, and represents an average for forty children.

TABLE III.
Showing the average time of forty pupils for each combination.
First Experiment.

Combination	Time	Combination	Time	Combination	Time	Combination	Time	Combination	Time
7+9	21.98	9+5	16.80	9+4	13.90	5+4	10.58	6+1	7.98
8+6	20.78	6+7	16.50	7+5	13.73	4+3	10.50	4+1	7.85
8+7	19.83	9+8	16.00	7+4	13.20	7+2	10.33	7+7	7.83
9+7	19.78	5+6	15.90	6+4	12.78	2+4	10.23	9+9	7.78
5+8	19.50	4+9	15.88	3+4	12.78	4+2	10.05	8+8	7.55
6+9	19.28	4+7	15.60	4+5	12.43	6+3	9.85	4+4	7.43
6+8	19.03	4+6	15.18	8+3	12.28	9+2	9.78	8+1	7.40
7+8	18.88	6+5	15.15	9+3	12.23	8+2	9.78	7+1	7.33
5+7	18.70	4+8	14.98	5+3	11.73	5+2	9.60	6+6	7.33
9+6	17.75	5+9	14.63	8+4	11.63	1+3	9.38	5+5	7.13
8+9	17.48	7+6	14.55	7+3	11.48	9+1	8.88	3+3	6.93
8+5	17.35					3+1	8.55	1+1	6.70
								2+2	6.20

Average time, 12.76

In Table IV the combinations used in the second experiment are given in the order of difficulty, the most difficult appearing first. This table is divided into six groups. In the first group the variation in response time is not more than three-fifths of a second. In the second, third, fourth, fifth, and sixth groups it is two-fifths, three-tenths, three-tenths, one-fifth and one-fifth, respectively. The time is again expressed in fifths of a second and represents an average for forty children.

TABLE IV.

Showing average time and order of difficulty for the forty pupils in the second experiment.

Combination	Time	Combination	Time	Combination	Time	Combination	Time	Combination	Time	Combination	Time
9+7	16.08	4+7	13.00	8+3	10.63	2+9	9.08	3+2	7.33	7+1	6.30
7+9	15.80	8+6	12.93	9+3	10.60	5+3	9.05	4+2	7.33	5+1	6.20
6+9	14.90	4+9	12.80	3+8	10.50	9+2	8.85	2+3	7.10	9+9	5.98
5+7	14.68	6+7	12.70	8+4	10.33	6+4	8.80	1+9	6.85	6+6	5.88
6+8	14.45	7+4	12.13	3+5	10.25	2+6	8.75	1+6	6.83	2+1	5.83
7+5	14.40	3+7	11.58	6+5	10.20	2+5	8.38	1+4	6.70	1+2	5.80
8+5	14.33	9+8	11.50	4+6	10.03	7+2	8.35	1+7	6.68	8+8	5.73
7+8	14.18	7+6	11.48	2+7	9.93	5+4	8.05	9+1	6.60	5+5	5.73
5+8	14.18	9+4	11.48	6+3	9.73	5+2	8.05	4+1	6.60	4+4	5.50
8+7	13.85	3+9	11.28	7+3	9.58	2+4	8.00	6+1	6.58	7+7	5.50
8+9	13.70	5+6	11.15	4+3	9.48	6+2	7.95	3+1	6.48	1+1	5.35
5+9	13.53	4+8	11.00	4+5	9.48	8+2	7.80	1+5	6.45	3+3	5.23
9+5	13.25			3+4	9.30			1+3	6.43	2+2	5.23
9+6	13.13			3+6	9.30			1+8	6.40		
				2+8	9.28			8+1	6.40		

Average of all, 9.48

In Table V the combinations are once more given in the order of difficult, the order being the result of a combination of Tables III and IV. The response times are given only for the combinations that are common to both. The other combinations have the same position in the series of Table V as in that of Table IV.

TABLE V.

Showing order of difficulty and average time used for combinations in both experiments.

Combination	Time	Combination	Time	Combination	Time	Combination	Time	Combination	Time	Combination	Time
7+9	18.89	4+9	14.39	7+4	12.67	2+9		3+2		9+9	6.88
9+7	17.93	4+7	14.30	4+6	12.61	4+3	9.99	4+2	8.69	5+1	
6+9	17.09	5+9	14.08	3+8		9+2	9.82	2+3		7+1	6.82
8+6	16.86	7+5	14.07	8+3	11.46	6+3	9.79	1+9		8+8	6.64
5+8	16.84	9+8	13.75	3+5		2+6		1+6		2+1	
8+7	16.84	3+7		9+3	11.42	2+5		1+4		1+2	
6+8	16.74	5+6	13.53	3+4	11.04	7+2	9.34	1+7		7+7	6.67
5+7	16.67	7+6	13.02	2+7		5+4	9.32	1+3	7.91	6+6	6.61
7+8	16.48	4+8	13.00	8+4	11.01	2+4	9.12	9+1	7.74	4+4	6.47
8+5	15.84	3+9		4+5	10.96	5+2	8.83	3+1	7.52	5+5	6.43
8+9	15.59	9+4	12.69	6+4	10.79	6+2		6+1	7.28	1+1	6.08
9+6	15.44	6+5	12.68	5+3	10.39	8+2	8.79	1+5		3+3	6.03
9+5	15.03			7+3	10.03			4+1	7.23	2+2	5.72
6+7	14.60			3+6				1+8			
				2+8				8+1	6.90		

Because its accuracy can be augmented by more statistical material, we intend to continue our study of the order of difficulty, the present publication being justified by the fact that the results already obtained have considerable value. To give some idea of the reliability of the results on the order of difficulty, we give in Table VI a comparison of the order found by the first study with that of the second. The order of the second study appears in groups as found in Table IV and that of the first is similarly grouped with blank spaces for the omitted combinations. There are only fifteen combinations which are not found in corresponding groups, and all but one of these are found in the contiguous groups. These fifteen combinations are followed by question marks in the table.

TABLE VI.

Comparing the order of difficulty of the second series of experiments with that of the first.

Second Study	First Study	Second Study	First Study	Second Study	First Study	Second Study	First Study	Second Study	First Study	Second Study	First Study
9+7	7+9	4+7	9+8	8+3	7+5?	2+9		3+2		7+1	9+9
7+9	8+6?	8+6	5+6	9+3	7+4?	5+3	5+4	4+2	5+2?	5+1	
6+9	8+7	4+9	4+9	3+8		9+2	4+3?	2+3		9+9	8+8
5+7	9+7	6+7	4+7	8+4	6+4?	6+4	7+2	1+9		6+6	4+4
6+8	5+8	7+4	4+6?	3+5		2+6		1+6		2+1	
7+5	6+9	3+7		6+5	3+4	2+5		1+4		1+2	
8+5	6+8	9+8	6+5?	4+6	4+5	7+2	2+4	1+7		8+8	8+1?
7+8	7+8	7+6	4+8	2+7		5+4	4+2?	9+1	1+3	5+5	7+1
5+8	5+7	9+4	5+9?	6+3	8+3	5+2	6+3?	4+1	9+1	4+4	6+6
8+7	9+6	3+9		7+3	9+3	2+4	9+2	6+1	3+1	7+7	5+5
8+9	8+9	5+6	7+6	4+3	5+3?	6+2		3+1	6+1	1+1	3+3
5+9	8+5	4+8	9+4	4+5	8+4	8+2	8+2	1+5		3+3	1+1
9+5	9+5			3+4	7+3			1+3	4+1	2+2	2+2
9+6	6+7?			3+6				1+8			
				2+8				8+y	7+7?		

The order of difficulty has considerable economic value. It enables the teacher to select with greater accuracy those combinations which require the most attention. She can more readily and surely avoid the waste which results from too great a neglect of the difficult sums and too frequent a repetition of the easy ones. From Tables IV and V she can get some notion of how much better some combinations are known that others. The most difficult combinations require a response time which is more than three times as long as that of the easiest. In the fifth and sixth grades we might have expected the children to know all of the additions so well as to preclude any large variations in response time. As this is not the case the teachers of these grades must still be concerned with teaching the simple combinations in addition, not to mention column addition. It is likely that the children in the sixth grade do not know the difficult sums any better than the third grade children know the easier ones.

A knowledge of the order of difficulty will also diminish the teacher's task of finding individual weaknesses, for evidently most of these are to be found in the most difficult groups. From our experience in making this study, we believe that an excellent method of finding individual shortcomings would be to take the child's response times for the combinations in two or three of the most difficult groups.

A number of interesting facts may be gathered from the preceding tables. Such combinations as 9+9, 8+8 and 7+7, have a shorter response time than 9+1, 8+1 and 7+1. The addition of 1 to a number is not so easy therefore as adding a number to itself. For this there may be several causes. As adding a number to itself gives the same result as multiplying by two, the children receive double drill on such combinations, once while learning to add and again while learning to multiply. Perhaps it takes a little longer to get 9+1 in mind clearly than 9+9 on account of a greater variety of mental imagery. Such combinations as 9+9 may also attract more attention because they are different from most combinations. Moreover in learning multiplication there is usually enough drill work of a kind that prevents roundabout methods of getting the results, and this is not always the case in learning additions. Where roundabout methods are employed the speed is of course diminished.

Other principles on establishing the order of difficulty of the combinations in addition may be found from a study of the tables. An analysis of Table V shows that out of the thirty-six combinations that were presented twice, once with the smaller number first and then with the larger number first, there are only five exceptions to the rule that it is more difficult to add two numbers when the smaller appears first in the combination. The exceptions are 9+5, 8+6, 8+7, 5+2 and 2+1. It may be easier to say 3 in response to 1+2 than to 2+1 on account of the habit of counting.

A selection from Table V of the order of difficulty for each digit in combination with every other digit, shows in general the truth of the statement that, in addition, the combinations which appear to be logically the most difficult are also psychologically the most difficult. In Table VII we give an arrangement for each digit with all the other digits, the most difficult combinations appearing first.

TABLE VII.

Arrangement of the order of difficulty of each digit with with every other digit.

9+7	8+6	7+9	6+9	5+8	4+9	3+7	2+7	1+9
9+6	8+7	7+8	6+8	5+7	4+7	3+9	2+8	1+6
9+5	8+5	7+5	6+7	5+9	4+8	3+8	2+9	1+4
9+8	8+9	7+6	6+5	5+6	4+6	3+5	2+6	1+7
9+4	8+3	7+4	6+4	5+3	4+5	3+4	2+5	1+3
9+3	8+4	7+3	6+3	5+4	4+3	3+6	2+4	1+5
9+2	8+2	7+2	6+2	5+2	4+2	3+2	2+3	1+8
9+1	8+1	7+1	6+1	5+1	4+1	3+1	2+1	1+2
9+9	8+8	7+7	6+6	5+5	4+4	3+3	2+2	1+1

Excepting the combinations with 1, it is possible to offer a plausible explanation for the order of almost all of the other combinations. The combinations with the highest numerical value are the most difficult. In the table there appear a few exceptions to this rule for which logical explanations can be made. Thus 9+8 is easier than the combinations that appear above it in the table because it is only one less than 2×9 and one more than 2×8; 8+7, 8+9, 7+6 and 5+4 are similar combinations which do not follow the rule of the highest numerical value. Further exceptions to the rule are furnished by the positions of 8+4, 4+8, 3+9 and 3+6. The addition of each of these pairs of numbers is facilitated because the larger number is an easy multiple of the smaller one. The only combinations whose positions remain unexplained are 8+7, 5+9, 3+8 and 2+9. More experimental data would undoubtedly show that they are incorrectly placed.

In the foregoing discussion we have endeavored to induct principles which would be of service in establishing a theoretical order of difficulty. These principles follow in the order of their predominance, the most predominant appearing first. Additions of equal numbers such as 6+6 are the easiest. This principle of course will not apply to the adding ability of children who do not know the simple multiplications by two. Second in point of ease are the combinations which have one very low number such as 1 or 2. Third, the combinations whose sum is low. Fourth, combinations whose sum is only 1 more or 1 less than twice either of the numbers added. Fifth, combinations in which one of the numbers is an easy multiple of the other. Sixth, combinations with the low number last are easier than the same combination with the low number first. The final position of this principle shall not indicate its relative importance in establishing an order of difficulty.

As the predominance of one principle over another is only in general true, their application in particular cases remains somewhat uncertain. While practically all of the equal number combinations are added more readily than combinations with one very small number, there may be exceptions. Perhaps 9+9 is more difficult than 2+1 as it appears in Table V because the addition of 2+1 involves the two principles of one very low number and a low sum. According to the principle of low sums 4+4 should be easier than 5+5, but perhaps this is not the case, because the children are in the habit of counting by fives. Moreover, the rule of low sums diminishes in importance among the difficult combinations, so that 9+5 and 8+6 appear to be more difficult than 8+9 even though their sums are smaller than that of 8+9. It is likely that among the more difficult combinations the third and fourth rules change their relative weight.

On account of these difficulties the accuracy of a theoretical order will be problematical. However, with the aid of the above rules and our experimental data, we have attempted to arrange a more satisfactory order of difficulty than those of the preceding tables. The order is given in Table VIII.

TABLE VIII.

Theoretical order of difficulty.

7+9	7+8	9+4	4+6	5+3	9+2	2+3	9+1	2+1
9+7	9+8	7+6	8+3	5+4	2+6	3+2	8+1	1+2
6+9	8+7	4+7	6+4	3+4	8+2	1+9	7+1	7+7
9+6	5+8	3+9	5+6	4+3	7+2	1+8	6+1	6+6
6+8	5+7	7+4	9+3	2+9	2+5	1+7	5+1	4+4
5+9	8+5	4+8	6+5	3+6	2+4	1+6	4+1	5+5
8+6	7+5	3+8	7+3	2+7	6+2	1+5	3+1	3+3
9+5	4+9	3+7	3+5	2+8	5+2	1+4	9+9	1+1
8+9	6+7	8+4	4+5	6+3	4+2	1+3	8+8	2+2

Among other interesting facts yielded by this part of our investigation are sex and grade differences. The combination 7+9 was found to be the most difficult for each of the sexes and for each of the fifth and sixth grades. The sexes differed in the length of their average response times, that of the girls being 2.13 fifths of a second less than that of the boys in the first series of experiments, but in the second series the time of the boys is less than that of the girls by .23 fifths of a second. An average for the two investigations would leave the response time of the girls shorter than that of the boys by .9 fifths of a second. The response time of the fifth grade is 4.30 and 1.32 fifths of a second longer than that of the sixth grade in the first and second experiments respectively. This gives an average excess of 2.81 for both experiments. In Table IX a few details on sex and grade differences are given.

TABLE IX.

(Fifths of second)	Boys	Girls	Grade — Fifth	Grade — Sixth
Average time for first series	13.82	11.69	14.91	10.61
Average time for second series	9.37	9.60	10.14	8.82
Average	11.60	10.65	12.53	9.72

For the second series of experiments the response times are much shorter than those of the first. The difference is primarily due to a change in the method of instruction which will be referred to again. The omission of some of the easier combinations from the first series may be mentioned as a minor cause. In the second series the average response time for all the combinations is 9.48, while that for the combinations used in the first series is 9.99, a difference of .51 fifths of a second.

In taking the children's oral responses we noticed that they were made much more quickly for some of the combinations than for others. We were interested to know whether the methods of adding for the slow responses differed from those for the quick responses; and, assuming that they did, what the nature of the differences was. These problems compose the second part of our study. The experimental work was carried out on two children from each grade from the third to the eighth inclusive. The experimental method was the same as that used to determine the order of difficulty with the exception that each child was asked to tell, immediately after his response, how he got the answer. The twenty combinations used were presented in the following order: (Read 4+3, 7+2, etc.)

4+3	7+2	2+4	4+4	8+5	7+4	6+9	3+4	5+5	6+8
5+3	7+8	6+6	7+9	4+8	7+5	4+7	9+9	7+6	6+5

Our results showed that many different modes of adding were used by the same child for different combinations and by different children for the same combination. The various modes that were used for 7+8, 6+9 and 4+7, which are representative, are given in Table X.

TABLE X.
Modes of adding.

7+8	6+9	4+7
Known	Known	Known
8+6+1	6+6+3	4+4+3
7+7+1	10+6—1	7+3+1
2×7+1	Guessed at	8+4—1
8+8—1	Counted 9, 10,11, etc.	Counted 4, 5, 6, etc.
Guessed at	Counted 6, 7, 8, etc.	Counted 7, 8, 9, etc.
Counted 7, 8, 9, etc.		Counted 7, 9, 11, etc.

The word **known** in the above table means that as soon as the child heard the combination, the sum occurred to him immediately without the intervention of any other process. The time for the known combinations was usually much shorter than the time for those whose sum was found by some other method.

In Table XI the tabulated results of our experimental work are shown. The letters A, B, C, D, etc., stand for the names of the pupils. The average time for each child's known combinations is expressed in fifth of a second, and that for the unknown is expressed in the same way. Unknown combinations are those whose sums were found in some roundabout way.

TABLE XI.

Time of known combinations compared with that of unknown.

Name	Grade	Number Known	Average Time	Number Unknown	Average Time
A.............	8	20	5.15	0	....
B.............	8	20	7.15	0	....
C.............	7	7	6.14	13	13.1
D.............	7	4	5.50	16	10.3
E.............	6	14	6.60	6	18.7
F.............	6	20	5.70	0	....
G.............	5	19	7.50	1	22.0
H.............	5	6	7.80	14	17.4
I	4	4	7.00	16	30.2
J	4	12	9.80	8	22.3
K.............	3	6	6.50	14	37.4
L.............	3	3	11.70	17	51.6
			Average, 6.38		Average, 22.3

It appears from the above table that the average time for the known combinations is only about one-third as long as the time for the unknown combinations. We believe therefore that children should be discouraged from using indirect methods of finding the sum of two numbers. Children should be told and required by methods of instruction to associate with two numbers, as 6+7, their sum 13. Methods of teaching which allow children to learn indirect methods of adding should be condemned. Indirect methods not only retard column addition, but also make the liability to error far greater. Here as in much other teaching we first allow or compel the children to form bad habits, and subsequently spend our time trying to break them of these undesirable habits. Indirect methods may at first give better objective results, but bad habits, loss of time, and final results must not be lost sight of in determining the best methods of instruction.

The third part of our study is a brief discussion of the value of drills. Before our first experiment on the order of difficulty, the children had received little or no drill in adding, but before the second experiment they had received daily three-minute drills for about a year. As the average response time of the fifth and sixth grades in the first experiment is 12.76 fifths of a second and only 9.99 fifths for the same combinations in the second, there occurred a reduction of 2.77 fifths of a second in the average response time. While this reduction is principally due to drill work, some of it must be ascribed to the fact that the second experiment occurred three months later in the school year than the first.

It is easy to calculate the amount of progress due to the drills from a knowledge of the progress in addition made by the fifth and sixth grades in a year. The progress of the fifth grade would be fairly well represented by the difference in response time of the two grades under consideration. On page 12 this was shown to be 2.81 fifths of a second. Assuming that the progress of the sixth grade would be the same (it would probably be less), a deduction of one-third of 2.81 or .94 would have to be made from 2.77 to find the amount of progress due to drills. As the remainder, 1.83, represents about two-thirds of a year's progress, we may say that the daily drills advanced the adding efficiency of the fifth and sixth grades by two-thirds of a year's progress.

The average yearly progress of the fifth and sixth grades in simple additions may be roughly calculated by using the Courtis standards and the response time of the fifth grade. According to these standards the progress of the sixth grade in the spring of the year will be eight fiftieths and that of the fifth grade eight forty-seconds of the response time of the fifth grade. These fractions should be increased to eight forty-sixths and eight thirty-eighths respectively, because yearly progress is a little more rapid in the lower than in the upper grades, our experiments having been made before and near the middle of the year. The average response time of the fifth grade as given in Table IX is 12.53 fifths of a second. By taking the average of eight forty-sixths and eight thirty-eighths of this time we obtain 2.41 fifths of a second as the average progress of the fifth and sixth grades for the year immediately preceding the time of our experiments. If 2.41 be accepted as a more accurate measure of the children's progress than 2.81, then the gain due to drills would be equivalent to about four-fifths of a year's progress. Even though our calculations are not flawless the fact remains that the children's adding ability was very much improved by the drills.

As a conclusion to the study we offer a few suggestions on the pedagogical side of the question, hoping that teachers who read this article will try them out and report results.

We think that very much more attention should be given to drill for the definite purpose of fixing the simple combinations so well that the reaction time will show little if any variation. It is no doubt the case that the combinations found in our most difficult group do receive more drill than the easy ones, but they should receive still more. The average time for the most difficult is about three times as long as the average time for the easiest. This is true of children who have received special drill on the harder combinations. It seems clear that the harder combinations, therefore, should appear in the drill exercises many times as often as they do now.

Children in the upper grades and many adults become so familiar with the elementary combinations that the reaction times show little variation. In one case in our last experiment we rejected the record of a sixth grade boy because the variations were so slight as to be almost negligible. His average time was 1.16 seconds; his mean variation was .52 seconds. His greatest variation occurred in 9+7, 4+9, 7+9, 5+8, 6+9, 9+4, 7+2, 9+3. If these are omitted the mean variation is .24 seconds. The average time of one of the poorest boys is 3 seconds, and his mean variation is 1.39 seconds.

We believe also that it is highly desirable to memorize the combinations so perfectly that the person on seeing or hearing the combination will instantly recall the sum. Objects or number pictures should be used in the perception of number and in developing addition in case of a few of the easy combinations. If continued too long they become a hindrance, so after presenting a few of the combinations by the use of objects, it is thought better to have the work done by the use of symbols only. If the pupil has forgotten the answer, he should be told, or allowed to look at a table containing the answers. Without such help he will resort to counting or some roundabout method. Roundabout methods are exceedingly wasteful. As shown by Table XI. the average time for the known combinations is about one-fourth of the average time for the unknown. There can be little doubt that hesitation and uncertainty become more pronounced in column addition and are a hindrance to accuracy.

We recommend that frequent tests be given in which the pupil is required to write or say the answers to the combinations so rapidly that he will not have time to use roundabout methods. The time should be recorded and the number of correct answer per minute should be noted. Individual weaknesses should be discovered and each pupil should contest against his own previous record. In many cases the pupils themselves will know their difficulties and will be glad to have the co-operation of the teacher in overcoming them.

To avoid errors, instruction in addition should begin with the simpler combinations and should introduce only one new combination with its reverse at a time. The new sums should be emphasized until they are well known. Then they may be placed on the review list where they should remain until they are permanently known. Nothing can be gained by introducing the new combinations too soon, or by dropping the old ones from the review list before they have been well fixed.