THE ROLE OF CALCULATORS IN DEVELOPING CHILDREN'S PROBLEM SOLVING SKILLS

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Abstract

This study investigated the role of calculators in the development of children's problem solving skills. Children from six second and third grade classes made up the two groups that were compared. The children and teachers in both groups were taught how to use calculators and had comparable calculator skills. The main difference was that one (experimental) group was exposed to more advanced problems in their math lessons, and the other group followed a standard curriculum. The experimental group substantially outperformed the standard group on a difficult test consisting of four word problems, which neither group had been given before, and which are usually given to children in grades five and higher. An analysis of the errors the children made showed that the difference did not lie in better arithmetic skills for the experimental group, but rather in their having a better understanding, and therefore a smaller number of conceptual errors. The study indicates that the use of calculators must be coupled with appropriate curricular changes in order to improve children's problem solving abilities.

The Role of Calculators in Developing Children's Problem Solving Skills

The National Council of Teachers of Mathematics, in its Curriculum and Evaluation Standards (1989), has mandated the use of calculators in schools from the earliest grades. Some states (e.g., Michigan; Michigan Council of Teachers of Mathematics, 1989) have also endorsed their use. As a result, calculators appear more and more often in classrooms. The discussion about calculator use centers mostly on childrens' arithmetic skills. This study, however, investigates the role that calculators play in developing children's problem solving skills.

Subjects.

One hundred thirty-five children from six classrooms (named A, B, C, D, E, and F) participated in the study. Classrooms A, B, and C were second grades and D was a 2/3 split class, from school one (these four classes comprised the whole second grade population at school one). Classrooms E and F were third grades from schools two and three respectively.

All schools were in small towns in rural areas. Schools one and two were from the same school district in the Midwest, and school three was in an adjacent school distict. There was no tracking in any of the schools, and the policy in both districts was to keep children with learning disabilities in regular classrooms. Only one classroom (F) had a significant number of minority (Hispanic) students (about 40%); and according to other teachers in school one, class D was "better than average" in that school.

Children in classrooms A, B, C. and D who attended school one the previous year as well as the current year were using calculators in their math classes for the second consecutive year. It was the first year that children in classrooms E and F were using calculators.

All six teachers attended at least four hours of workshops on using calculators in elementary mathematics. The workshops were led by the first author of this paper. The teachers were given an opportunity to meet on a regular basis with the first author, and she gave at least one demonstration lesson in each teacher's classroom.

Curricula in the classrooms.

Teachers in the six classrooms selected the math instruction for their classes on an individual basis. There was a significant difference in the curricula followed in the different classrooms.

In classrooms A and B the teachers elected to follow the standard second grade curriculum (the textbook they used was by Orfan, Vogeli, Krulik, & Rudnick (Silver-Burdett, 1987)). Their children used calculators occasionally as an enrichment, and more regularly to check the results of paper and pencil computations. The remaining four classes primarily followed an experimental curriculum. (Information about the frequency of use of calculators and about the types of activities the teachers did with them comes mainly from their own regular reports to the authors of this paper.)

Experimental curriculum involving calculators.

The teachers were given a large selection (over 40 lessons) of project-like problems. (Examples of problems include measuring and calculating the perimeter of a polygon in centimeters and millimeters, and checking a real four-item grocery bill from a supermarket (involving multiplication of decimals).) In order to do the tasks in the lessons, children often had to perform complex computations requiring many operations. A typical problem would take one class period, but some took more. The kind of mathematics occurring in the experimental lessons varied greatly. Some problems even required the extraction of square roots. Teachers selected the problems that they wanted, and gave them in the order of their choice. They also requested lessons on specific topics, and these were especially prepared for them. (In one such lesson, children constructed a Valentine heart and computed its area.) Teachers spent one to two periods per week (on average) on these projects, and they used the remaining time allotted for mathematics instruction to teach the standard curriculum for second (or third) grade. Short word problems, such as those given below, were not part of the experimental material, and were used only in testing.

Test.

Late in the spring semester children were given four word problems:

1. John and Mary bought for lunch one hamburger for $1.75, one cheeseburger for $1.95 and two colas for 62 cents each. How much did they pay?

Answer: $4.94

2. Fifteen children went on a trip to the zoo. The teacher bought an ice cream cone for each child and for herself. One ice cream cone costs 48 cents. How much did she pay?
Answer: $7.68

3. Sue has two quarters, one dime, four nickels, and three pennies. How much money does she have?
Answer: $0.83, or 83¢.

4. Three people paid $14.50 for lunch. They decided to share the cost equally. How much should each person pay?
Answer: $4.83 or $4.84

Children received the problems as a typed handout, but teachers also read the problems aloud to the children.
We note that these problems are not typically presented to children in second and third grade. They occur more traditionally in fifth grade or higher textbooks.

Results.
The answers were read by at least two people, and if there was a disagreement about what a child had recorded, a third person was consulted. We first report the number of numerically correct answers (Table 1). (An answer was numerically correct if it was correct without regard for units, e.g, dollars and cents. If the correct answer contains a decimal point, then in order to be numerically correct, the recorded answer must also contain a decimal point.)

Table 1. Number of numerically correct answers, by problem, by classroom, and by standard vs. experimental curriculum.

Problem no.		1 menu	2 ice cream	3 coins	4 sharing bill	total
Class	(no. of children)					
A	(21)	1	2	2	1	6 (7%)
B	(24)	2	0	3	0	5 (5%)
Total:		3 (7%)	2 (4%)	5 (11%)	1 (2%)	11(6%)
C	(23)	13	10	9	7	39 (42%)
D	(22)	7	7	11	11	36 (41%)
E	(25)	6	9	11	1	27 (27%)
F	(20)	8	5	13	0	26 (32%)
Total:		34 (38%)	31 (34%)	44 (49%)	19 (21%)	128 (35.6%)

Some conclusions:

- We see that all of the problems were too difficult for the children who were learning the standard curriculum (those in classes A and B); they scored 5 to 7% correct.
- Children experiencing the experimental curriculum solved approximately 1/3 of the problems.
- It was not arithmetic per se that made the problems difficult. The only one-operation problem (number 4) happened to be the most difficult, and the problem that requires the most operations (number 3) received the most correct answers.
- One should not think that the differences in the performances in a particular classroom are attributable only to differences in the mathematics curriculum. Those teachers who take part (voluntarily) in an experimental mathematical program usually like the topic and are more enthusiastic about teaching mathematics. We also know that children who are participating in the

experimental curriculum form positive attitudes toward math, and therefore probably put more effort into solving the problems they are given.

Table 2 shows the number of children who got 0, 1, 2, 3, or 4 answers correct.

Table 2. Number of children who got 0, 1, 2, 3, or 4 answers correct, by classroom, and by standard vs. experimental curriculum

Number of answers correct:	0	1	2	3	4
A (21)	16	4	1	0	0
B (24)	19	5	0	0	0
Total:	35 (78%)	9 (20%)	1 (2%)	0	0
C (23)	7	5	3	4	4
D (22)	3	7	7	5	0
E (25)	9	9	3	4	0
F (20)	5	5	9	1	0
Total:	24 (27%)	26 (29%)	22 (24%)	14 (16%)	4 (4%)

More conclusions.

- We see that in the experimental classrooms, C, D, E, and F, 73% of the children solved at least one problem correctly, and 44% solved at least two problems. This shows that good problem solving skills are learnable, and that they are accessible to most, if not all, children, not only to a talented few.

What kinds of errors do children make?

Calculators do not guarantee correct answers. During complex computation it is easy to miss a key, to press a wrong digit, or to make other errors of this kind. Are children making errors because of a lack of typing skill? Or do they make errors because they lack mathematical knowledge? Or do they make an incorrect

Are children making errors because of a lack of typing skill? Or do they make errors because they lack mathematical knowledge? Or do they make an incorrect match between mathematics and the real world? We will show that most of the errors come from a lack of understanding, and not a lack of skill.

Error analysis.

A calculator display can show up to 8 digits, so it often happens that a wrong answer recorded by a child leaves an informative "footprint," namely, the error the child made can be reasonably deduced from the answer he or she recorded. (Ashlock (1994) analyzes errors children make in hand computation.)

Examples.

In the first problem (menu; its correct answer is $4.94), the wrong answer that occurred most often was 4.32 (this answer occurred 16 times). This occurs when .62 is added only once rather than twice. Sixteen children ignored that there were *two* colas, costing 62 cents *each*. These results are similar to those of Cummins, Kintsch, Reusser, & Weimer (1988). They found that first, second, and third graders' errors on word problems were often correct solutions to miscomprehended problems, and that word problems with abstract or ambiguous language were miscomprehended more often than those using simpler language. (See also Cummins (1991); Davis-Dorsey, Ross, & Morrison, (1991); Riley & Greeno, (1988); and Sowder (1988).)

In problem number 4 (sharing three ways a bill for $14.50; the correct answer is $4.83 or $4.84), five children gave 0.28 as the answer, and four children gave 64. The first number (0.28) is obtained when 14 is divided by 50, and the second is 14 + 50.

Many children treated the decimal point as a separator between two different whole numbers. After all, $5.45 is read, 5 dollars *and* 45 cents, and 12.3 cm is read, 12 centimeters *and* 3 millimeters. So in problem 4, children who correctly chose division as the operation and saw 14.50 as two separate numbers made the first error. And children who used the common strategy, "When in doubt, add," made the second error.

In problem 3 (coins; correct answer is 83¢), the answer 33.5 was probably obtained as follows: Two quarters are .50; a dime, four nickels and three pennies are 33; so the total is 33.5. The answer was most likely copied from the calculator display (which shows 33.5 rather than 33.50), because otherwise a child would probably write 33.50.

Table 3. Errors on problem 1, by classroom.

Problem 1 (menu); correct answer is $4.94.
Code of errors (number of times it occurred):

#1 correct answer (37 of 135 total answers)

#2 4.32 or 4.35; only one cola counted (17)

#3 65.7; only one cola counted, and it is counted as 62 and not 0.62 (6)

#4 answers in range $2 to $9 (38)

#5 answers out of this range (34) or no answer recorded (3)

Code	#1	#2	#3	#4	#5
A (21)	1	0	1	4	15
B (24)	2	0	4	8	10
C (23)	13	0	0	6	4
D (22)	7	2	0	11	2
E (25)	6	8	1	5	5
F (20)	8	7	0	4	1
Total	37	17	6	38	37

Comments.

- Consider error number 2 above, in which children recorded 4.35. Children often read '2' as '5' from the display; on the calculator, '2' and '5' are mirror images. Errors number 2 and 3 occurred 23 times, or 17% of all 135 answers that were given to problem one. This is 23% of all the errors in problem one.
- The 'answer' 127.7, in which there is no decimal point in front of 62, and two colas are counted (the button presses would be [1.75][+][1.95][+][62][+][62][=]), did not occur at all.
- Most of the errors in groups #4 and #5 involved some wrong decision (the errors were not purely arithmetical). The main difference was that #5 usually involved a mishandling of the decimal point.
- Children in the standard math program (in classrooms A and B) had less experience with decimals than children in the experimental program. This is

understandable, because most of the instructional time in second grade mathematics is spent on whole numbers. A lack of experience with decimals in classrooms A and B is evident from columns #3 and #5, where answers with large numbers were often recorded because children misplaced the decimal point.

Problem 2 (ice cream); correct answer is $7.68.
Code of errors (number of times the error occurred):

#1	correct (33)
#2	7.20, teacher's ice cream cone was not included (12), or 48 or 0.48, teacher paid for herself only (7)
#3	64, 63, and 16, adding 15 + 1 + 48 and so on (17)
#4	no answer (11)
#5	other answers (55), contain a large variety of errors. For example two children gave the answer 16.48, sixteen people, 48 cents; and one child wrote 15.48

Code	#1	#2	#3	#4	#5
A (21)	2	2	7	2	8
B (24)	0	4	3	7	10
C (23)	10	1	1	0	11
D (22)	7	0	2	2	11
E (25)	9	3	3	0	10
F (20)	5	9	1	0	5
Total	33	19	17	11	55

Comments.

- This problem is most directly solved as a multiplication problem; however, children can solve it by addition using the constant feature of the calculator (by pressing 48 + and = = = ... sixteen times).

The number of incorrect multiples of 48 (17 of the errors in column #5 were of this type) indicates that this strategy was used, but that the children lost count in pressing =.

This shows that better arithmetic skill, or understanding that multiplication can replace repeated addition, would add 17 items to columns #1 and #2.

- #3 and many errors in group #5 show indiscriminant addition, often with no regard for meaning.
- The number of times that no answer was recorded, versus the number of times a wrong answer was given, in both this and the remaining problems, reflects different teachers' classroom policies more than childrens' knowledge. Some teachers encourage guessing, while others say, "If you do not know the answer, do not write anything."

The total number of instances in which no answer was recorded, in each classroom:

A	B	C	D	E	F
2	26	0	3	0	1

Problem 3 (coins); correct answer is \$0.83, or 83¢.

Code for errors:

#1 correct

#2 conceptual errors or no answers

There were not many clear conceptual errors; two children said 10, which is the number of coins; 5 children gave 43 = 25 + 10 + 5 + 3, or 41 = 25 + 10 + 5 + 1; and 7 started listing the coins, 2510... . Three children did not give an answer.

#3 value close to the correct answer.

#4 values above \$1, indicating errors with decimals, such as 50 + .10, etc.

Code	#1	#2	#3	#4
A (21)	2	7	7	5
B (24)	3	7	13	1
C (23)	9	1	3	10
D (22)	11	1	6	4
E (25)	11	1	7	6
F (20)	13	0	6	1
Total	49	17	42	26

Comments:

- In this problem the third grade classes (E and F) and the 2/3 split class (D), performed the best. Children were not given any manipulatives to work with. We think that if they were given real coins, the results would be very different, namely, the younger children would perform substantially better.
- Groups A and B made the most conceptual errors (#2).

Problem 4 (sharing a bill); correct answer is $4.83 or $4.84.

Code for errors:

#1 correct
#2 misplaced decimal point: 483.33333 or 48.333333
#3 value estimated, probably by trial and error
#4 wrong operation: addition, subtraction, or multiplication
#5 operating on two numbers, 14 and 50
#6 other errors
#7 no answer

Code:	#1	#2	#3	#4	#5	#6	#7
A (21)	1	0	0	2	7	11	0
B (24)	0	0	1	1	3	5	14
C (23)	7	4	0	2	2	8	0
D (22)	11	1	2	4	0	4	0
E (25)	1	0	5	6	2	11	0
F (20)	1	0	5	5	4	4	1
Total	21	5	13	20	18	43	15

Comments

This is the only problem that cannot be solved by addition. Therefore, it provided the biggest variety of errors.

- The relatively good performance in classrooms C and D shows that these children had one or more lessons that involved division. In the experimental curriculum, some problems require division, and experienced teachers used some of them in the second grade (and even in the first grade).
- Column #3 shows those children who tried general problem solving techniques and came close to the answer. For example, "One person pays $5 and the others pay $4.50 each."
- Column #4 shows a rather expected error: "Pressing one of the keys will give an answer; now I have to guess which one." It is possible that a few (correct) answers in #1 were the result of a lucky guess.
- Column #5 is the most disturbing. It shows that many children have a deep misunderstanding of decimals, and that they treat a decimal point as a separator between two whole numbers. This misunderstanding is probably far more prevalent than 18/135 = 13%, because in many cases (in #6) it could have been masked by other errors.
- The difference between classrooms A and B is simple. Children in both groups did not know how to solve the problem, and they probably knew that they could not. In classroom B they simply skipped it, and in classroom A they tried to do something.
- No student noticed that 4.83 times 3 is 14.49 and not 14.50.

The use of units, namely, dollars and cents, in the four problems.
The number of children who:

#1	correctly used units in one or more problems, and made no errors in their use;
#2	made errors in usage of units:
#3	did not use units at all.

Code:	#1	#2	#3
A (21)	1	0	20
B (24)	3	5	16
C (23)	0	1	22
D (22)	8	9	5
E (25)	20	3	2
F (20)	5	7	8
Total:	37	25	73

Comments:

- With one exception (classroom E), the children either did not write units or wrote them incorrectly. Typical errors were .83¢ or $4.93¢, which indicate that children followed a spoken pattern and treated the decimal point as a marker for cents. This table also shows that the proper use of dollar and cent signs was not taught in five out of six classrooms.

Conclusions.

The data from all groups show that giving children calculators and simply teaching them how to use them is not enough.

Explanation.

With some simplifications, finding the solution of a word problem that requires calculations consists of the following steps (a model for solving geometry problems is given by Polya (1957)):

(1) Understanding the problem.

Examples:

- Two colas were bought.
- All the children and the teacher had ice cream.

(2) Translating the problem into mathematical terms.

Examples:

- If $1.95 is 1.95, then 62 cents is 0.62 and not just 62.
- Three people are dividing a bill equally. Sharing calls for division.

(3) Planning the computation, namely choosing the operations and numbers on which they operate.

Examples:

- Add 0.48 sixteen times.
- Multiply 0.48 by 16.
- Divide 14.50 by 3.

(4) Doing the actual calculations (executing the plan).

Examples:

- Enter .48, press [+], and press [=] 16 times.
- Enter .48, press [x], enter 16 press [=].

(5) Evaluating and checking the solution.

Examples:

- I got an answer of $64 for ice cream. That is too much! I made an error.
- I got 83. It means 83 cents, and not 83 dollars.

Skill in using a calculator helps mainly in step (4). But the error analysis shows that most systematic errors can be traced to steps (1) and (2), and that they go undetected because of a failure in step (5).

Errors that can be solely attributed to a failure in steps (1) and (2):

#2 in problem 1 (only one cola was counted),
#2 and #3 in problem 2 (e.g., teacher's ice cream cone was not included),
#2 in problem 3 (e.g., counting the number of coins), and
#4 and #5 and probably most of #7 in problem 4 (e.g., wrong operation, or operating on the two numbers 14 and 50).

That point (5), checking the answer, was not followed, is seen from the number of answers that do not fall within a reasonable range, anywhere close to the correct answer. It is clear that children are not accustomed to reevaluating their numerical solutions. This is confirmed by a systematic omission of units of measurement. In finances, the answer 83 is meaningless if we do not know if it is $83 or 83¢.

Errors due to poor use of the calculator did not seem to play a big role for children in the experimental curriculum. The hardest problem arithmetically was problem 3 (coins), which required 13 to 19 keystrokes, depending on the method used.

Examples of sequences of keystrokes:

25+25+10+5+5+5+5+3= (19 keystrokes)

25+==+10=+5====+3= (18 keystrokes)
25[M+][M+]10[M+]5[M+][M+][M+][M+]3[M+][MRC] (18 keystrokes)
Additional keystrokes could have been needed for clearing the memory and the display. More experienced children could have simplified the computation by doing part or all of it mentally: 50+10+20+3=.

The number of incorrect answers in problem 3 that can be attributed solely to arithmetic errors is at most 42 (#3, indicating errors that are incorrect but close to the correct answer), which is smaller than the number of correct answers, 49 (#1).

Problem 3
Number of children who got the correct answer (#1)
and whose answer was incorrect but close to correct (#3)

	#1	#3
A and B together (45 children total):	5	20
C, D, E, and F (90 children total):	44	22

The table shows that 2/3 of the children in the experimental classes who avoided conceptual errors were successful with arithmetic. This is in contrast to paper and pencil calculations, where arithmetic errors are very common, even in very simple problems. The small rate of success (5 of 45 correct) among the children taking the standard curriculum is probably due to the fact that, although they were using calculators regularly, they did not have much experience with computations that require several operations. (Such problems are rare in the second grade.)

Final words.
Systematic large differences in performance between children taking the experimental curriculum and children taking the standard one show that childrens' problem solving skills can be improved dramatically if the use of calculators is paired with appropriate curricular changes.

The main role of calculators is that they allow teachers to give a large variety of interesting and challenging problems, and to switch the stress from teaching the mere mechanics of computation to using mathematics as a tool in solving real life problems.

The test problems discussed above were especially created to cause many errors that can be studied. During regular classes children have very high rates of success solving problems from the experimental curriculum. That success is a very strong confidence builder, especially for children having low skill in paper and pencil computations.

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