DO CHILDREN REQUIRE DRILL IN ORDER TO ACQUIRE SKILL IN USING CALCULATORS

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Abstract

The question we pose is, can children attain proficiency in using calculators without drill, but rather in the context of meaningful problem solving? Children in five second grade classes used calculators in their math lessons at least once a week for two semesters. There was no drill and practice with calculators. Late in the second semester the children were tested on five problems in addition of numbers with decimal points. Nine of the 97 children missed all problems; 38 got 4 or 5 correct; and 50 got 1 to 3 correct. Those who got at least one problem correct we categorized as either competent in using calculators for complex addition or on their way to competence -- if they made errors, we could classify the large majority as errors in correct procedures. We discuss in detail our error analysis, and also the reasons for differences in performance.

Do children require drill in order to acquire skill in using calculators?

Introducing calculators in the early elementary grades, which has been mandated by the National Council of Teachers of Mathematics (1989), and strongly encouraged by the NCTM 1992 yearbook, brings with it several questions. One question, which is not the topic of this paper, is how calculator use will influence children's learning of mathematics. Another is whether using calculators in early math instruction will lead to children's competence in performing everyday arithmetic calculations. And a third is whether such competence, if it can be attained, can be achieved without drill, but rather in the context of meaningful problem solving.

Tenth grade students who were allowed for the first time to use calculators on Michigan's state-wide MEAP (Michigan Assessment of Educational Progress) test did not seem to be helped significantly by using them (Payne, 1992, reporting pilot-test data). Barely 1/2 of the students, given the fraction 51/68 and given a calculator, could express the fraction as a percent (on a four-alternative multiple choice test); and less than half could compute 4.56 x 18/25. Our experience with teaching competent adults how to use four-operation calculators[1] indicates that it takes at least 12 hours of training to became practically familiar with their essential features and several weeks of actual use before a satisfactory level of performance (error-free to a high degree) is achieved. Teachers who introduce calculators for the first time into their classrooms have reported that in the beginning students make many errors in even very simple calculations, and this often leads to disappointment and discouragement for both themselves and their students. Some teachers try to remedy this situation by requiring drill in the use of calculators. This is a very unattractive solution. Replacing hand calculation drill by calculator drill misses the main purpose of using calculators, which is to free children from mindless, boring, constant practice (Baggett & Ehrenfeucht, 1992; Grossnickle, Perry, & Reckzeh, 1990).

The question which we asked in this study, and which we can now (tentatively) answer, is the third one posed above: Can children achieve high competence in using calculators without drill?

Ninety-seven children from five second grade classes in two different schools in rural south-central Michigan towns participated in the study. There was no tracking into classes in either school; children of all abilities were present in each class. These five classes were the entire population of second graders in the two schools.

Each classroom had a set of four-function calculators, one calculator per child, supplied by the school district (all were Texas Instruments 108 calculators). Calculators were used for mathematics instruction in each classroom once or more per week throughout the year. In three of the classes (in school one), calculator lessons were mostly led by the teachers, although at least one demonstration lesson using calculators was given in each class by one of the authors of this paper. Teachers in school one had a total of at least 12 hours of workshops in calculator use and were meeting weekly with one of the authors. In the other two classrooms (in school two) calculator lessons were led approximately once a week by one of the authors of this paper, who came in as a guest instructor. The teachers in these classes were less experienced with calculators; they had had only one workshop and several other shorter sessions. But they occasionally gave calculator lessons in their classrooms, especially as follow-ups to the lessons given by the guest instructor.

In all five classrooms calculators were being used on a pilot basis. So, while they were used at least once a week, most of the children's math lessons did not involve calculators. Teachers in school one used the Silver-Burdett second grade math book for most of their lessons (Orfan, Vogeli, Krulik, & Rudnick, 1987), and teachers in school two used the Heath series second grade math book, Heath Mathematics Connections (Manfre, Moser, Lobato, & Morrow, 1992).

In the calculator lessons, children were involved in solving meaningful problems; there was no drill and practice. So there were no calculator problems such as, "Add these five numbers:...," and no calculator worksheets. Each class had at least one calculator lesson in which children made measurements of distances in centimeters and millimeters, and answered various questions involving lengths of paths which required addition of numbers with decimal points. And all children had at least one calculator lesson involving a menu. They calculated such things as the cost of 4 hamburgers and 2 cokes, when a hamburger costs $1.19 and a coke 85 cents. Some teachers reported using variants of the menu lesson as many as five times during the year, and most teachers also used calculators for checking one or more grocery receipts. So all children had been introduced to decimal notation with lessons about money and about centimeters and millimeters.

The children's knowledge of decimal notation was very limited. Most of them were rather competent in handling positive integers up to 1000, but their knowledge of decimal fractions was concrete. In the context of money they treated the decimal point as a separator between the number of dollars and the number of dimes and pennies. Most were able to read correctly $3.05 as three dollars and no dimes and five pennies, and $3.5 as three dollars and five dimes, namely, three dollars and fifty cents. But they read the calculator display of 0.3333333 (obtained by dividing 1 by 3) as "thirty three cents and a little bit more." In the context of length, they treated the decimal point as a separator between centimeters and millimeters, but some children already knew that .5 is "how the calculator shows 1/2" and were able to read 3.5 as "three and a half centimeters." Some could read 3.15 as "three centimeters and one and a half millimeters." The children did not know expanded notation, or any underlying principles of decimal notation. For example, even if they knew that 10 millimeters make one centimeter, they did not know that one millimeter is 1/10 of a centimeter. In general, their concept of fractions was limited to 1/2, 1/4, and 1/3.

Late in the second semester the children were given a test in addition of numbers with decimal points. The test, consisting of five problems, and its answers, are given in Appendix 1. The children were given the test without any meaningful context. They were simply told that they should add the numbers in each problem and write down the answer. The reader may notice that only the first problem can be reasonably interpreted as adding a list of prices in dollars and cents. The purpose of the exercise was not to test their understanding of numbers in decimal notation, but to test their skill in adding a list of numbers using a calculator, independent of their meaning.

Findings.

The answer for each problem given by each child was read by at least two people. If there was doubt about what the child had written (in about 15 of the 485 answers), the two readers settled on one answer after a discussion or asked a third reader for an opinion in order to reach a final decision. The children's answers were entered into a data file for computer analysis. As shown in Table 1, two hundred fifty-eight answers (53.2%) were correct. Sixteen answers (3.3%) were left blank. And 211 answers (43.5%) contained errors, namely, the answers recorded did not match the correct answers. (Figure 1 shows test sheets from two children.)

Table 1

Total percent correct, percent with no answer, and percent with answers we could and could not classify; and number of children (out of 97) who got correct 0, 1, 2, 3, 4, or 5 problems.

Of the 485 possible answers from all children (97*5):

percent correct	53.2
percent with no answer	3.3
percent with answers we couldn't classify	14.4
percent with answers we could classify	29.1

number of problems correct	number of children
0	9
1	9
2	17
3	24
4	21
5	17

Name: Jacob

12.30	23.9	1234.76
3.58	1.23	9804.8
78.34	20.56	0.7835
0.90	128.7	75300.23
4.65	0.1	98.0750
99,77	174,49	86438,46

9.87, 7.654, 3.213, 4.852 sum = 25589

84765.38, 154, 236, 0.76 sum = 85156,14

Name: Ryan

12.30	23.9	1234.76
3.58	1.23	9804.8
78.34	20.56	0.7835
0.90	128.7	75300.23
4.65	0.1	98.0750
99.77	174.49	86438.648

9.87, 7.654, 3.213, 4.852 sum = 25.589

84765.38, 154, 236, 0.76 sum = 8156.14

Figure 1. Test sheets from two children.

Table 2

Percent correct, percent with no answer, and percent with answers we could and could not classify, in the group of 9 children who missed all problems and in the remaining group of 88 children.

	9 children	88 other children
number of possible answers	9*5=45	88*5=440
percent correct	0	58.6
percent with no answer	7	3.0
percent with answers we couldn't classify	89	6.8
percent with answers we could classify	4	31.6

A first finding (see Tables 1 and 2) was that the group of 97 children divided into two parts. Nine children (9.3%) did not get any of the five problems right, and they mostly gave nonsensical answers (e.g., one child wrote the answer to problem 5 as 152; the correct answer is 85156.14). Clearly these children didn't have any idea how to use a decimal point, or didn't have any idea how to use a calculator for simple calculations, or both. We were able to classify errors (see below) in only two of the problems from these nine children (about 4%; see Table 2). Three of their 45 answers were left blank. So we could not classify errors for 40 (89%) of their answers. In the remaining group of 88 children, there was a rather uniform spread of from one to five problems correct (see Table 1), and in this group there were only 30 problems out of 440 (less than 7%) whose errors we could not classify (see Table 2).

Table 1 shows that 17 children (17.5%) got a perfect score of 5 problems correct. They clearly achieved proficiency in using a calculator for addition of decimals. Twenty-one more got 4 correct out of 5. Nine got zero out of 5, and 50 were somewhere in between (they got 1, 2, or 3 out of 5). This performance occurred without drill.

Special consideration should be given to the group of nine children who did not get any of the five problems correct. At this stage, clearly the calculator did not help them get a correct answer. We know that the reason for this is not that these children were especially learning disabled, because some learning disabled children performed surprisingly well on the test, with scores that put them among the top half of their classmates. We noticed that 5 of the 9 children were in one of the five classrooms (see Table 3). During the year their teacher had been frequently absent because of illness, and they developed serious disciplinary problems. So it is possible that the scores of these children simply reflect a lack of participation and a lack of attention to activities during the year.

Table 3

Test outcomes of 88 children (data from the 9 children who missed all problems have been removed*)

class-room	no. of kids	total no. probs.	no. of probs correct	problems with no answer	problems with unclass. errors	problems with class. errors
1	17(2)	85	54	2	4	25
2	16(1)	80	55	0	6	19
3	17(1)	85	55	0	7	23
4	22(0)	110	52	11	5	42
5	16(5)	80	42	0	8	30
tot.	88(9)	440	258	13	30	139

*In parentheses are the number of children in each class who missed all problems.

Method of error analysis: Keystroke and transcription errors.

The question examined in this section is how to classify errors that the children made. When a child wrote an answer that did not match the correct answer given on the calculator display, we knew that something went wrong. Our task was to determine what went wrong. There were two kinds of errors we looked for: keystroke errors (errors in pressing keys) and transcription errors (errors in reading the display or in copying from it).

First we looked for keystroke errors. The diagnosis of these errors for each problem was based on a comparison of two procedures (two sequences of keystrokes). The first procedure gives the correct answer to the problem, and the second gives the actual answer written down by a child. The keystrokes in one procedure were matched with those in the other in order to indicate the mismatches. The wrong answer could then be attributed to a specific mismatch.

We were not present when the tests were administered and keystrokes were not recorded in any way, so we do not have information on what children actually did while solving the problems. There are many possible procedures which yield a given answer, but only some classes were considered as candidates, as follows.

We assumed that a correct procedure was addend + addend + ... + addend =, where addends could be entered in any order. So a correct procedure for a problem with addends a, b, c, and d is [a] [+] [b] [+] [c] [+] [d] [=]. Note that any procedure (such as [a][+][b][=][+][c][=][+][d][=], or [a][M+][b][M+][c][M+][d][M+][MRC]) which gives the correct answer cannot be distinguished from the procedure we assumed.

We also considered only cases in which the calculator was cleared between problems, or in which the last operation in the preceding problem was [=]. If the actual keystroke sequence a child pressed was different, e.g., if the child used [M+] and did not clear memory between problems, such an error was not detected at all and would fall into our "unclassifiable" category. Besides these, we looked for one other entry error: accidentally using [-] instead of [+]. No such error was found.

For example, problem 1 on the test is:

12.30
3.58
78.34
0.90
4.65

The correct answer is 99.77; the answer given by a child was 454.19. We first give a correct sequence, with a sequence yielding the recorded answer directly below it. The discrepancy between the two is marked with a ^.

12.30 + 3.58 + 78.34 + 0.90 + 4.65 =

^

12.30 + 358 + 78.34 + 0.90 + 4.65 =

We hypothesize that the child omitted the decimal point from the second addend.

For every correct procedure, a specific possible set of procedures containing keystroke errors was considered first. If an answer written by a child could be explained by a keystroke error or errors, it (or they) was (were) classified as the probable cause of the error. If, as mentioned above, an answer could not be explained by an entry error, we looked for a second kind of error, one in reading from the display or writing the answer. These we called transcription errors, and we looked for them because in many cases all digits in an answer were correct, but the decimal point was missing from the answer. So the decimal could not have been omitted from the addends (except in problem 1). Here is an example (problem 4): 9.87, 7.654, 3.213, 4.852 sum = ______. The correct answer is 25.589, and one answer recorded was 25589, with no decimal point. This answer occurred seven times in our data.

If an error could not be explained by a keystroke error and could be explained by a transcription error, then it was classified as a transcription error. Table 4 describes the four kinds of keystroke errors and the two kinds of transcription errors we looked for.

Table 4

Description of keystroke and transcription errors. We looked for the following deviations from a correct keystroke sequence, as defined in the text:

abbreviation:	description of keystroke error
DA	Error in which a decimal point is missing from one addend
KA	Error in which a key is mispunched (or a digit is misread) in an addend (excluding decimal point errors)
JORA	Structural error in which two addends are joined (because of omission of [+]); or one or more addends are omitted or repeated.
OA	Other: shift of decimal point in addend (e.g., 9804.8 (problem 3) typed as 980.48) or block change in addend (e.g., 75300.23 typed as 75730.23 (problem 3))

We looked for the following transcription errors:

abbreviation:	description of transcription error
digit	reading or writing error involving a digit (and not a decimal)
dec	reading or writing error in which decimal is missing

Examples of procedures used in error diagnosis.

Diagnosing keystroke errors was done with the help of an interactive program. Here are some procedures that were used: (1) We looked for an error of omission of the decimal point in an addend as follows. When there were five addends, there were five different cases to consider, namely, answers obtained when the decimal point was omitted from each addend in turn. We looked at the results one would get from these five cases, and if a result matched an answer recorded by a child, we said it was likely that the child omitted the decimal point from that particular one of the five addends. (2) We looked for an error consisting of mispunching one digit as follows. For each digit in each addend, there were nine possible mispunches. We tried them, and looked at the results. If one matched a child's answer, we said it was probably the case that the child mispunched the specific digit which yielded the result. (3) We looked for errors consisting of permuting two consecutive digits in an addend by trying all the instances, and if one matched a child's answer, we said that the child probably made this particular permutation. (We did not look for permutations of 3 or more consecutive digits.)

The method used in searching for errors of the types listed in Table 4 was always successful if a wrong answer could be accounted for by just one error. But when we increased the number of errors that we looked for in one problem, the chance of being able to classify them decreased very fast, so that only in a very few cases were three or more errors in one problem detected. Most unclassified problems (except for those made by the nine children discussed above) are probably due to an accumulation of simple errors. When multiple explanations were found that would account for an answer, the answer was left unclassified.

The procedures used for error diagnosis (for both keystroke and transcription errors) allow us to say only that a particular (wrong) answer recorded by a child could have been caused by the mechanism we found that yields the answer. We cannot guarantee that the child actually made the error that we hypothesized. But the hypotheses about the errors children made were based on what we knew, from our own classroom observations and from teachers' reports, about how children generally perform calculations. And it was not often that two different errors would lead to the same result. For example, one error that was found was omission of a decimal point from an otherwise correct answer. It was classified as omission of a decimal point in transcription (copying from the calculator display to the paper.) We do not know any systematic error in entering numbers, i.e., any keystroke error, that would yield this result, except in problem 1, where omission of decimal points from all addends would match the trascription error.

Specific errors that were found.

We restrict our findings to errors made by the 88 children who correctly answered one or more problems. Table 5 gives examples of each type. There were 196 such errors, as shown in Table 6. Most of these errors (152, or 78%) were made in entering symbols (see Table 6). The remaining 44 (22%) were made in reading and writing multidigit numbers. Sixty-four percent of the transcription errors (28 of 44) consisted of not writing down the decimal point (see Table 6). This is an important finding: those children who had the answer correct (except for writing the decimal point) correctly entered the decimal point in their keypunching, but they didn't pay attention to it in writing their answer. We think these were conceptual errors: the concept of decimal point in the abstract really had no meaning for these children. They ignored it because they didn't see a need for it. (Omission of the decimal point in an otherwise correct answer was classified not as a keypunch error but as a transcription error.) It was mentioned earlier that at the time of the study, the children had had little experience with decimal notation; they knew it in the concrete (with money and centimeters and millimeters), but not in the abstract.

Table 5

Examples of errors and possible explanations for them. The caret symbol ^ indicates a mismatch in a correct procedure (given first) and a hypothesized incorrect procedure (given second).

```
 Keystroke errors:

 DA (decimal point missing in an addend)
     problem 1. answer given: 454.19; correct answer: 99.77
     12.30 + 3.58 + 78.34 + 0.90 + 4.65 =
              ^
     12.30 + 358  + 78.34 + 0.90 + 4.65 =
     Explanation: The addend 3.58 was typed as 358.

     problem 4. answer given: 4872.737;  correct answer:
     25.589
     9.87 + 7.654 + 3.213 + 4.852 =
                             ^
     9.87 + 7.654 + 3.213 + 4852 =
     Explanation: The addend 4.852 was typed as 4852.

 KA (single entry error)
```

```
    problem 2. answer given: 74.49; correct answer 174.49.
    23.9 + 1.23 + 20.56 + 128.7 + 0.1 =
                          ^
    23.9 + 1.23 + 20.56 +  28.7 + 0.1 =
    Explanation: The addend 128.7 was typed as 28.7.

    problem 2. answer given: 173.59; correct answer 174.49
    23.9 + 1.23 + 20.56 + 128.7 + 0.1 =
       ^
    23   + 1.23 + 20.56 + 128.7 + 0.1 =
    Explanation: The addend 23.9 was typed 23.

JORA (join, omit, repeat addend)
    problem 3.  answer given: 85203.888; correct answer
    86438.648
    1234.76 + 9804.8 + 0.7835 + 75300.23 + 98.0750 =
    ^^^^^^^^^
              9804.8 + 0.7835 + 75300.23 + 98.0750 =
    Explanation: The first addend, 1234.76, was omitted.

    problem 1. answer given: 98.8709; correct answer 99.77
    12.30 + 3.58 + 78.34 + 0.90 + 4.65 =
                         ^
    12.30 + 3.58 + 78.34   0.90 + 4.65 =
    (this is registered as
    12.30 + 3.58 + 78.34090 + 4.65 =.)
    Explanation: The plus sign between the addends 78.34 and
    0.90 was omitted, so that the two numbers were joined
    to make 78.34090

    problem 3.  answer given: 13607.938; correct answer
    86438.648.
    1234.76 +  9804.8 + 0.7835 + 75300.23 + 98.0750 =
                                 ^^^^^^^^
               9804.8 + 0.7835 + 98.0750 + 1234.76 = = =
    Explanation:  first addend was repeated 3 times; 4th
    addend was omitted (change of order is also
    hypothesized).

OA (other)
    problem 3. answer given: 77614.328; correct answer:
    86438.648
    1234.76 + 9804.8 + 0.7835 + 75300.23 + 98.0750 =
                 ^^
    1234.76 + 980.48 + 0.7835 + 75300.23 + 98.0750 =
    Explanation: decimal point in 9804.8 shifted to
    980.48:

    problem 3.  answer given: 76633.888; correct answer
    86438.648
    1234.76 + 9804.8 + 0.7835 + 75730.23 + 98.0750 =
    ^^^^^^^ ^ ^
               804.8 + 0.7835 + 75730.23 + 98.0750 =
```

```
    Explanation: 1234.76 omitted; 9804.8 typed as 804.8

Transcription error:

digit (reading/writing error involving digit; different
from omitting decimal)
problem 5. answer written: 85,156.14;
                              ^
           answer from display: 85156.14.
           Child added comma to answer.

Keystroke and transcription error:

problem 3.  answer given: 85327.6; correct answer 86438.648
1234.76 + 9804.8 + 0.7835 + 75300.23 + 98.0750 =
   ^
 123.76 + 9804.8 + 0.7835 + 75300.23 + 98.0750 =
          Explanation:  1234.76 typed as 123.76; did not
          write last 2 digits of answer
```

Table 6

Number of instances of each type of error from the 196 classifiable errors from 139 problems given by 88 children.

error type	number of instances	%
(1) keystroke errors:	152	78%
DA (decimal pt. missing in addend)	24	
KA (mispunched key in addend; not dec pt)	85	
JORA (addends joined, omitted, repeated)	39	
OA (other; see Table 5)	4	
(2) transcription errors:	44	22%
dec (dec pt omitted from answer)	28	
digit	16	
tot. no. of keystroke&transcription errors	196	

Among keystroke errors categorized in Table 5 as joined, omitted, or repeated addends, two types that were found which would never occur during hand calculation should be mentioned. One is joining two consecutive numbers, due to the omission of [+]. (Note that in column addition with at least 3 addends given in textbooks, at most one plus sign is typically shown, so this error could be a carryover from hand addition.) The other is duplication of an addend due to the repetition of [=]. (Note that, while pressing [+] twice in a row has the same effect as pressing it once, duplicating [=] has the effect of adding the most recently entered addend. So pressing a function key twice sometimes has side effects and sometimes doesn't.)

Misuse of function keys (other than [=] and [+]) was not detected. It was mentioned that we looked for accidental use of [-], so it would have been detected if it had occurred. But accidental use of other function keys ([√], [*], [÷]) was not looked for and therefore would not have been detected if it had occurred; it would lead to unclassified errors. But if such errors occurred, they did not occur often, because the total number of unclassified errors (30) among the 88 children was not very large.

Tables 7, 8, and 9 show the data by problem. Table 7 shows that problem 3 was the most difficult in terms of number of keystrokes to press and number of symbols to write. Tables 8 and 9 show that it was also the most difficult in terms of percentage correct (42%) and total number of errors (59 of 196 errors, or over 30%). Table 8 shows that the number of problems with no answer is 0, 0, 1, 4, and 8, for problems 1,2,3,4, and 5 respectively. This could indicate children ran out of time for problems nearer the end of the test.

Table 7

Characteristics of the problems: number of keystrokes to press and number of symbols to write (assuming constraints on keystroke entry as discussed in text)

problem number	number of digits	number of dec. points	number of plus signs	number of equals signs	number of symbols to write in answer
1	17	5	4	1	5
2	16	5	4	1	6
3	29	5	4	1	9
4	15	4	3	1	6
5	16	2	3	1	8

Table 8
Number and percentage correct, number with no answer, and number with unclassified errors, by problem, for 88 children.

problem number	number correct	% correct	number with no answer	number unclass-ified
1	63	72%	0	5
2	51	58%	0	3
3	37	42%	1	11
4	59	67%	4	6
5	48	55%	8	5
total	258		13	30

Table 9
Number of errors of each type made by the 88 children, by problem.

problem	keystroke errors:					transcript. errors:			tot
number	DA	KA	JORA	OA	tot	dec	digit	total	errors
1	6	9	6	0	21	2	2	4	25
2	7	23	6	1	37	2	6	8	45
3	5	24	15	3	47	4	8	12	59
4	2	4	1	0	7	3	9	12	19
5	4	25	11	0	40	5	3	8	48
total	24	85	39	4	152	16	28	44	196

Discussion

There appear to be three different phenomena going on with the 97 children: (1) Most children (81 of 97) try to answer all the problems; they basically know what they are doing, and they are rather successful at doing it. (2) Some children (9 of 97) write answers that are nonsensical. They do the task (provide answers to the problems), but they don't seem to know what they are doing. They punch some keys and write something. (3) Some children don't do some of the tasks--they omit answers. This phenomenon is somewhat special. One would expect that the children who aren't able to do the task at all (those described in (2)) would be the ones who would leave answers blank. But only one child is in this category: this child gave two answers which we could not classify and left the other three blank. There were seven other children who left one or more answers blank, and they made a total of only 2 unclassifiable errors.

We really do not know what causes children to write nonsensical answers or to omit answers. We have given possible reasons for the writing of nonsensical answers by one group of children: there was a lack of discipline in their classroom. For children in category (3), it could be that some wanted to avoid errors, or it could be that there was lack of time to do the test. We note that children in each of the categories (2) and (3) were found primarily in one particular classroom (see Table 3). And both were found primarily in school 2, where calculator activities were taught mostly by a guest lecturer. The teacher in class four, containing the largest number of children who left answers blank, mentioned that her children "did not want to make errors" because the guest instructor would know about them.

A question is whether the group of 88 children were (a) executing an incorrect procedure, or (b) making errors in a correct procedure. The errors we diagnosed are clearly in the second category, e.g., a decimal point is missing in just one addend. But it is possible that the errors we did not diagnose are due to an incorrect procedure, for example: (1) omitting a decimal point (always) from all addends; or (2) pressing [=] several times instead of once; or (3) systematically omitting 0 from each number. Any such incorrect procedure would appear as an unclassified error.

It is possible that the nine children who missed all problems were using incorrect procedures. But we think it is more likely that their errors were due simply to random typing rather than anything systematic, because, as mentioned before, 5 of the 9 were from a classroom in which the teacher had missed many classes and there were serious discipline problems.

Brown and vanLehn (Brown & VanLehn, 1980; VanLehn, 1982) suggested reasons for children's errors in performing multidigit subtraction by hand. For the most part, they claimed that the children they studied were correctly executing an incorrect procedure. Children in this study, however, are doing something different. Using VanLehn et al.s' terminology (e.g. vanLehn, 1980, p. 12) our students' errors are "slips" (small errors in an otherwise correct procedure), and not "bugs," (perturbations indicating that the procedure itself is incorrect). Further, our children were not drilled in addition with a calculator, so we think they are making soft (unstable) errors. On the other hand, children in the VanLehn studies, we think, had harder (more stable) errors, since they had been drilled.

We see three differences between the results for calculations done by hand and those done with a calculator:

1. The calculator procedure is easier.
2. The calculator procedure is more transparent -- the reason for each step is more obvious.
3. Because of lack of drill, we think that our children were confused and did not know what to do, rather than displaying an established bad (incorrect) habit.

Reviewing again the results, seventeen of 97 children reached competence (got 5 out of 5). Twenty-one almost reached it (got 4 out of 5). Fifty still need to improve considerably (got 1, 2, or 3 out of 5); and nine didn't get any answers correct. Eighty-eight of the 97 children were either competent in using calculators for complex addition, or on their way to becoming competent in their use -- if they made errors at all, the large majority were errors we could classify, namely, errors in correct procedures. We think that these findings answer the question posed at the beginning. Skills in arithmetic (with a calculator) CAN be achieved by solving meaningful problems, and not spending time on pages of worksheets.

A question is, what kinds of lessons should be given to the children who are incorrectly carrying out correct procedures, in order to help them improve their skill in using a calculator? What type of material should be used? We think that, based on the actual training that the children had, it is more a question of attention and care focused on the task than a question of skill: children need to learn to look at the calculator display after they press keys. This is different from simply practicing when a skill is nearly automatic, in order to make it automatic. Children need to learn to pay careful attention to details in pressing, and to look at the results of a press. At this point, we suggest for these children more of the same kinds of lessons with calculators that they have been getting already in their classrooms: not drill and practice, but involving meaningful problem solving.

Over the years many researchers (e.g., Brueckner, 1930; Ashlock, 1976; Brown et al, 1980; VanLehn, 1982; vandeWalle, 1990, p. 164) have analyzed errors people make in hand calculation . But analysis of errors made in calculator computation is new. We were able to account for a large percentage of the errors children actually made, using fairly straightforward methods involving only six classes of simple errors.

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Appendix 1.
Test given to children
(Note: 14-point font was used. See also Figure 1.)

Name: ______________________________

12.30	23.9	1234.76
3.58	1.23	9804.8
78.34	20.56	0.7835
0.90	128.7	75300.23
4.65	0.1	98.0750

9.87, 7.654, 3.213, 4.852 sum = __________

84765.38, 154, 236, 0.76 sum = __________

Answers to problems as given on calculator display: 99.77; 174.49; 86438.648; 25.589; 85156.14.

Footnote

[1]In the last three years one of the authors has given hundreds of hours of workshops on calculator use to several hundred teachers in grades K-9.